Chapter 5
Unsteady Flow

5-1. Introduction

This chapter is presented in two sections. Section I presents guidance on the practical use of unsteady flow modeling and Section II presents some theoretical considerations regarding various routing techniques. Guidance regarding the application of unsteady flow models is presented first because the theoretical information, although important, is of a more general nature.

Section I
Application of Unsteady Flow Models

5-2. Steady versus Unsteady Flow Models

The traditional approach to river modeling has been the use of hydrologic routing to determine discharge and steady flow analysis to compute water surface profiles. This method is a simplification of true river hydraulics, which is more correctly represented by unsteady flow. Nevertheless, the traditional analysis provides adequate answers in many cases. This section identifies when to use unsteady flow analysis.

a. Steady flow. Steady flow analysis is defined as a combination of a hydrologic technique to identify the maximum flows at locations of interest in a study reach (termed a "flow profile") and a steady flow analysis to compute the (assumed) associated maximum water surface profile. Steady flow analysis assumes that, although the flow is steady, it can vary in space. In contrast, unsteady flow analysis assumes that flow can change with both time and space. The basics of steady flow analysis were given in Chapter 2; details may be found in Chapter 6.

(1) The typical steady flow analysis determines the maximum water surface profile for a specified flood event. The primary assumptions of this type of analysis are peak stage nearly coincides with peak flow, peak flow can accurately be estimated at all points in the riverine network, and peak stages occur simultaneously over a short reach of channel.

(2) The first assumption allows the flow for a steady state model to be obtained from the peak discharge computed by a hydrologic or probabilistic model. For small bed slopes (say less than 5 feet per mile), or for highly transient flows (such as that from a dam break), peak stage does not coincide with peak flow. This phenomenon, the looped rating curve effect, results from changes in the energy slope. The change in slope can be caused by backwater from a stream junction, as shown in Figure 5-1, or by the dynamics of the flood wave, as depicted in Figure 5-2. Since coincidence of peak stage and flow does not exist in either of these cases, the proper flow to use in a steady flow model is not obvious.

(3) The second assumption concerns the estimation of peak flow in river systems. For a simple dendritic system the flow downstream from a junction is not necessarily equal to the sum of the upstream flow and the tributary flow. Backwater from the concentration of flow at the junction can cause water to be stored in upstream areas, reducing the flow contributions. Figure 5-2 shows the discharge hydrographs on the Sangmon River at the Oakford gage and at the mouth of the Sangmon River 21 miles downstream. The outflow hydrograph is attenuated and delayed by backwater from the Illinois River. Steady state analysis often assumes a simple summation of peak discharges. For steep slopes, once again, the assumption may be appropriate but its merit deteriorates as the gradient decreases.

(4) A more difficult problem is that of flow bifurcation. Figure 5-3 shows a simple stream network that drains a portion of Terrebonne Parish in Louisiana. How can the flow in reach 3 be estimated? Figure 5-1 shows the hydrograph at mile 0.73 in reach 3; note the flow reversal. Hydrologic models and steady state hydraulics cannot predict that division of flow or the flow reversals.

(5) The third assumption allows a steady flow model to be applied to an unsteady state problem. It is assumed that the crest stage at an upstream cross section can be computed by steady flow analysis from the crest stage at the next downstream cross section; hence, it is therefore assumed that the crest stage occurs simultaneously at the two cross sections. Because all flow is unsteady and flood waves advance downstream, this assumption is imprecise. As the stream gradient decreases and/or the rate of change of flow increases, the looped rating curve becomes more pronounced, and the merit of this assumption deteriorates.

(6) The three assumptions are usually justified for simple dendritic systems on slopes greater than about 5 feet per mile. For bifurcated systems and for systems with a small slope, the assumptions are violated and the

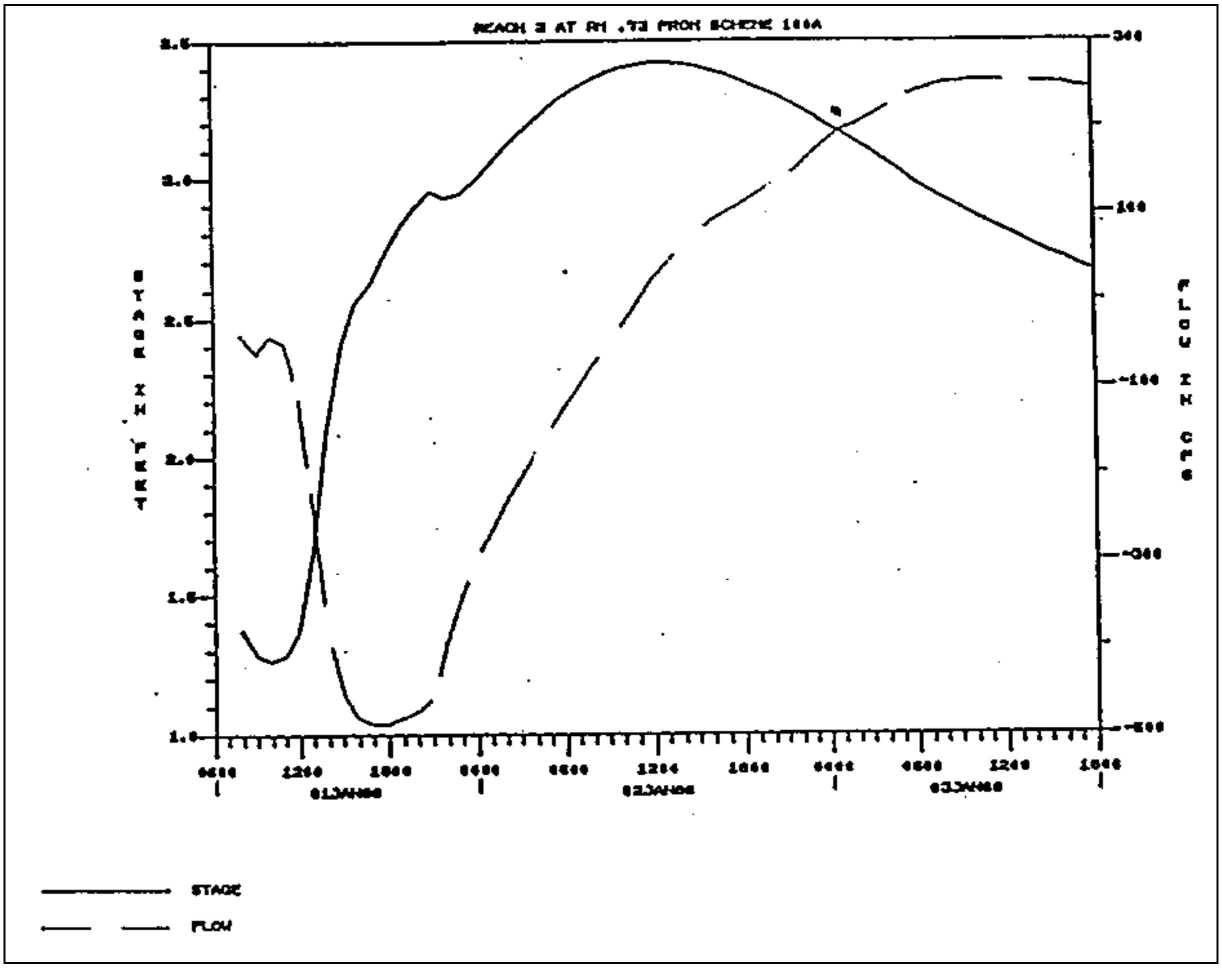

Figure 5-1. Looped rating curve induced by backwater

profiles from a steady flow model are suspected. In general, for large rivers and low lying coastal areas, steady flow analysis is not appropriate.

5-3. Conditions that Require Unsteady Flow Analysis

Unsteady flow analysis should be used under the following conditions:

a. Rapid changes in flow and stage. If the inflow or the stage at a boundary is changing rapidly, the acceleration terms in the momentum equation (see Section 5-12) become important. The leading example is dam break analysis; rapid gate openings and closures are another example. Regardless of bed slope, unsteady flow analysis should be used for all rapidly changing hydrographs. Any information on events of record, high water marks, eyewitness accounts, and so on can be useful in identifying such conditions. Eyewitness accounts of the Johnstown dam-break flood, for example, describe seiching in a major tributary valley. Occupants of floating houses made the trip up and down the valley several times as the currents reversed direction. Only an unsteady flow model with all acceleration terms intact is capable of modeling such an effect on downstream hydrographs and water levels.

b. Mild channel slope. Unsteady flow analysis should be used for all streams where the slope is less than 2 feet per mile. On these streams, the loop effect is predominant and peak stage does not coincide with peak flow. Backwater affects the outflow from tributaries and storage or flow dynamics may strongly attenuate flow; thus, the profile of maximum flow may be difficult to

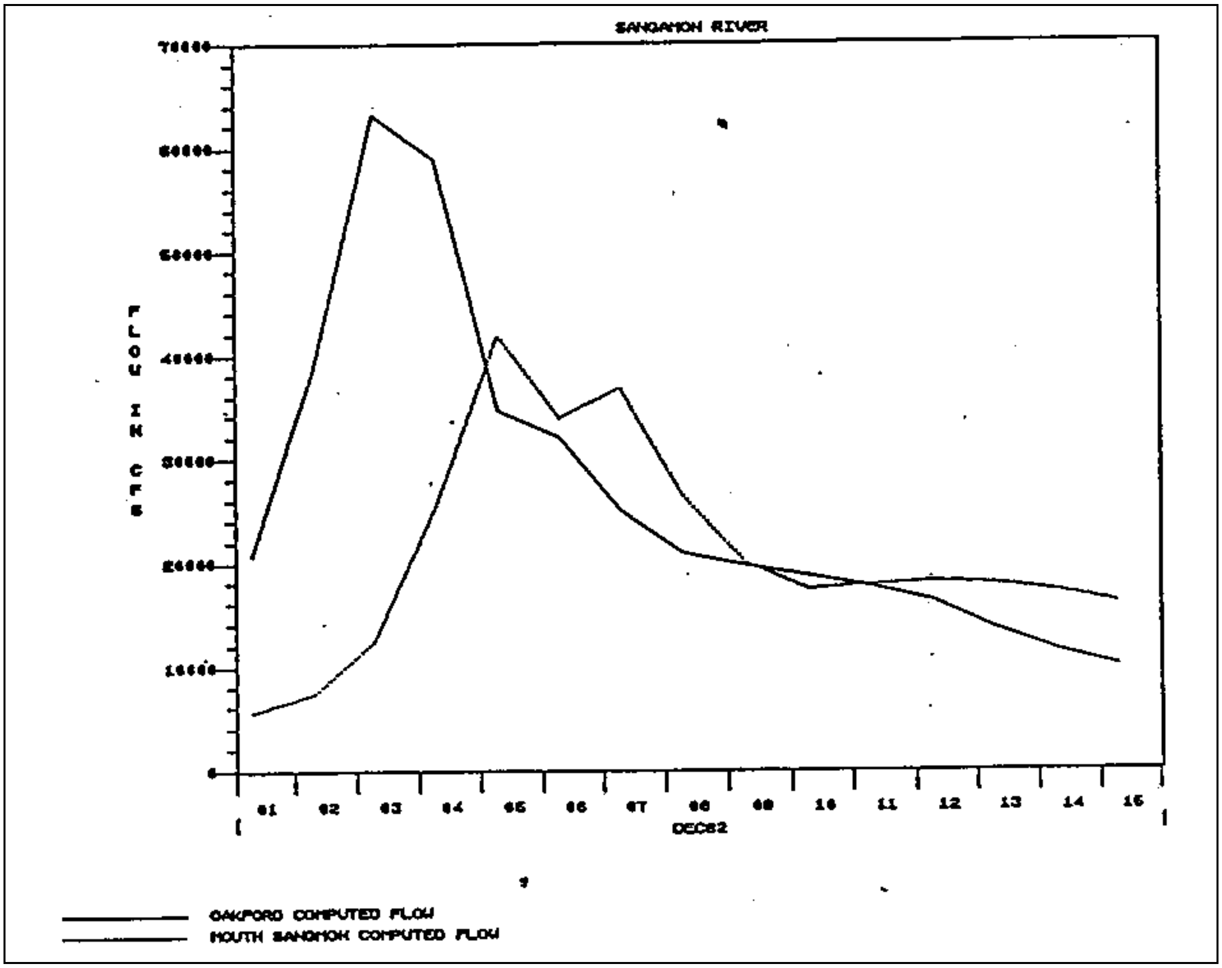

Figure 5-2. Discharge hydrograph at the oakford gage at the mouth of the Sangmon River

determine. For bed slopes from 2 to 5 feet per mile, the need for unsteady flow analysis may depend upon the study objectives. Large inflows from tributaries or backwater from a receiving stream may require the application of unsteady flow. Flow reversals may occur under such conditions, rendering hydrologic routing useless. For slopes greater than 5 feet per mile, steady flow analysis is usually adequate if the discharge is correct.

c. Full networks. For full networks, where the flow divides and recombines, unsteady flow analysis should always be considered for subcritical flow. Unless the problem is simple, steady flow analysis cannot directly compute the flow distribution. For supercritical flow, contemporary unsteady flow models cannot determine the split of flow. Records of current speeds and directions at different points in a flooded valley and rates of inundation of floodplains help determine whether a one-dimensional approach to a simulation is adequate (see Chapters 4 and 6).

5-4. Geometry

The geometry of the reach can be determined from topographic maps, surveyed profiles and cross sections, onsite inspection, and aerial mapping.

a. Costs. The influence of errors in reach geometry on predicted stages can be estimated based on regression equations given in "Accuracy of Computed Water Surface Profiles" (U.S. Army Corps of Engineers 1986). Profile errors can also be investigated in a simplified, though representative, reach by modifying its geometry in accord with the possible error and noting the effect on predicted discharges and stages. The costs associated

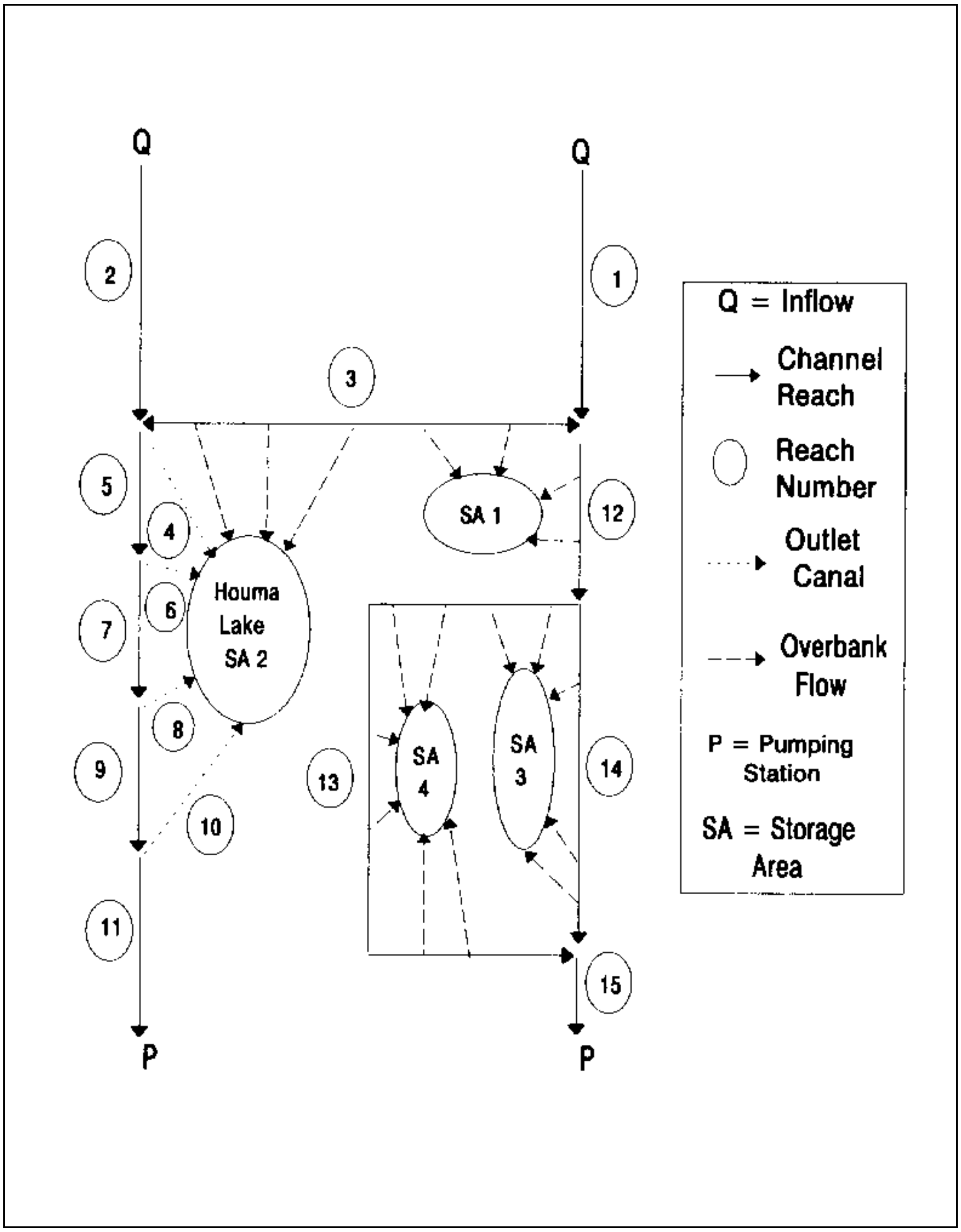

Figure 5-3. Network of flow system at Terrebonne Parish near Houma, Louisiana

with surveys of various degrees of accuracy can be estimated from "Accuracy of Computed Water Surface Profiles" (U.S. Army Corps of Engineers 1986, 1989).

b. Changes. Visual inspection of a reach must be done to identify the nature of the boundary material, vegetation, and human activities. Alluvium is subject to scour and deposition with possibly major changes in cross section shape accompanying a major flood event. As gross changes in cross section occur within alluvial streams, roughness also changes as dune patterns change during a flood event. Estimated changes in roughness can be applied to a rigid bed model to evaluate the importance of their effect. Prediction of boundary movement lies outside the scope of this Chapter; refer to Chapter 7 and EM 1110-2-4000.

c. Micro-geometry. Visual inspection should be used to identify the boundary roughness and other characteristics, such as potential infiltration, of a reach. Infiltration is usually of concern for overland flow; occasionally however, significant water loss (or gain) from a channel will occur in sand, karst, or volcanic geology. Boundary roughness affects the passage of a flood wave. Inspection of the study reach will indicate the nature of the roughness elements: cobbles, boulders, trees, houses, their density and distribution, and variance of roughness with stage and distance down the reach. First approximation values for roughness parameters can be gleaned from past experience with similar roughness elements (Chow 1959, Chapter 5); the drag of trees, and small structures can be estimated from expected velocities, areas of projection normal to the expected flow, and an estimated drag coefficient. Improved values of roughness are obtained by comparing computed and observed flows and stages for events of record.

5-5. Controls

a. Hydraulic controls. Hydraulic control sections should be sought out because these are natural reach delimiters. At such a section, there is a unique stage-discharge relation (except for the hysteresis induced by unsteady flow), unaffected by flow conditions downstream; hence, it is ideal for a gaging station. It is possible that a control is weak; that is, a rising downstream water level can drown the control section and force its effect upon the subject reach. In that case the reach cannot be studied independently of downstream reaches. This possibility can be investigated with steady flow analyses based on projected flood discharges.

(1) The issue of downstream control is significant to the choice of flood routing method. Influences on water levels within a reach stemming from conditions downstream (tidal levels, or increased levels due to small slope, high roughness, or flow constrictions downstream, for example) preclude application of hydrologic methods. Known water levels (say, tidal) at the downstream end of a reach allow use of hydraulic methods. Otherwise the downstream boundary must be extended until a control (or known level) is encountered.

(2) Downstream from a critical depth control is supercritical flow. If the channel downstream is hydraulically steep and sufficiently long to encompass the reach of interest, supercritical flow will persist all the way down the reach. No independent downstream boundary condition is possible, since downstream depth and discharge are dictated by the flow in the reach. The correct way of modeling such a flow is with an unsteady flow model. If available models cannot deal with supercritical flow, a diffusion model will yield a reasonable result if water surface elevations are not needed and the stream is not extremely flat.

(3) In most cases, the zone of supercritical flow is relatively short, ending either in a plunge into a pool of subcritical flow or joining subcritical flow downstream in a hydraulic jump. In unsteady flow, this jump (called a hydraulic bore) can move about.

b. Friction control. A so-called friction control pertains to a section in a nearly uniform reach, sufficiently long to insulate the section from downstream backwater. Then, the stage-discharge relation is governed by a condition of normal depth (near normal in the case of unsteady flow). This type of downstream boundary condition is well suited for all flood routing techniques that recognize downstream boundary influences.

5-6. Boundary Conditions

"Boundary conditions" is a mathematical term which specifies the loading for a particular solution to a set of partial differential equations. In more practical terms, boundary conditions for an unsteady flow model are the combination of flow and stage time series, which when applied to the exterior of the model either duplicates an observed event or generates a hypothetical event such as a design flood, or dam break. For an observed event, the accuracy of the boundary conditions affects the quality of the reproduction. In a similar but less detectable manner the reasonableness of the boundary conditions for a hypothetical event (because accuracy can seldom be established) limits the quality of the conclusions. Furthermore, the way that the boundary conditions are applied can control the overall accuracy and consistency of the model.

a. Upstream boundary conditions. The upstream boundary condition defines an input to be routed through the system. In most cases this is either a flow or stage hydrograph.

(1) Flow hydrograph. A flow hydrograph is the classic upstream boundary condition where the time varying discharge is routed downstream and the corresponding stages are computed by the model at the upstream boundary and elsewhere. If the flow hydrograph is at a gaging station, the location of the station should be checked. If the station is on a stream with a flat bed slope or with a highly mobile bed, a stage boundary condition may be preferable for reproducing an observed event. However, the flow boundary may be acceptable if the upstream boundary is on a smaller tributary which only makes a minor contribution to the overall system. For this case any error would be lost in the overall system. Be careful when using flows from a slope station as an upstream boundary condition; the values may not be accurate, resulting in an inability to calibrate.

(2) Stage hydrograph. If a stage hydrograph is used as an upstream boundary, the corresponding flow is computed from the conveyance given by the geometric data. Because errors in stage data are less than errors in flow data, the stage hydrograph may have substantial advantages in accuracy over the flow hydrograph. The stage hydrograph is used when a flow station is not available or the quality of flow data is in question. Flow computed from a stage boundary must always be verified against reliable flow measurements, otherwise substantial error in flow can result. If no flow measurements are available, the stage hydrograph should only be used when absolutely necessary and then with caution. Figure 5-4 shows the reproduction of flow measurements at Hickman from routing Cairo stages down the Mississippi River. Figure 5-5 shows the reproduction of stage at Memphis 200 miles downstream.

b. Downstream boundary condition. For subcritical flow, the downstream boundary condition introduces the effect of backwater into the model. Four types of downstream boundary conditions are stage hydrograph, flow hydrograph, rating curve, and Manning's equation.

(1) Stage hydrograph. The classic downstream boundary is the stage hydrograph. The corresponding flow is calculated by the model. Because the stage hydrograph is observed, and therefore presumed accurate, the downstream end of a study reach can be located at a gage.

(2) Flow hydrograph. The flow hydrograph is a special purpose downstream boundary condition which is generally used to simulate a reservoir outflow or a pumping station if accurate outflow is known. For the flow hydrograph, the model calculates the corresponding stages. The time series of computed stages is based on an initial stage and will change with a differing initial stage. The flow hydrograph must be used with great care because the flow is leaving the system and negative depths may be computed, in particular at pumping stations.

(3) Rating curve. A single valued rating curve describes a monotonic relationship between stage and flow. The rating curve is accurate and useful for describing a boundary condition at a free overfall, such as a spillway or at a falls, or at a pump station whose performance is defined by a schedule. But the single valued rating curve is often a poor downstream boundary condition for a free flowing stream. Natural rivers display a looped rating curve; use of a single valued rating curve, however, forces a monotonic relationship which erroneously reflects waves upstream. For this reason, the rating curve must be located well downstream of the reach of interest in a free flowing stream to prevent errors from propagating upstream into the area of interest. This lack of sensitivity should be confirmed by sensitivity tests.

(4) Manning's equation. Manning's equation can be used as a downstream boundary condition for a free flowing stream when no other boundary condition is available. The model computes both stage and flow with the stage being a function of the friction slope. Two methods prevail for determining the friction slope. Fread (1978, 1988) in DWOPER and DAMBRK assumes that the friction slope is equal to the water surface slope.

UNET (U.S. Army Corps of Engineers 1991b) uses the friction slope at the last cross section. These two assumptions, which produce slightly different results, are

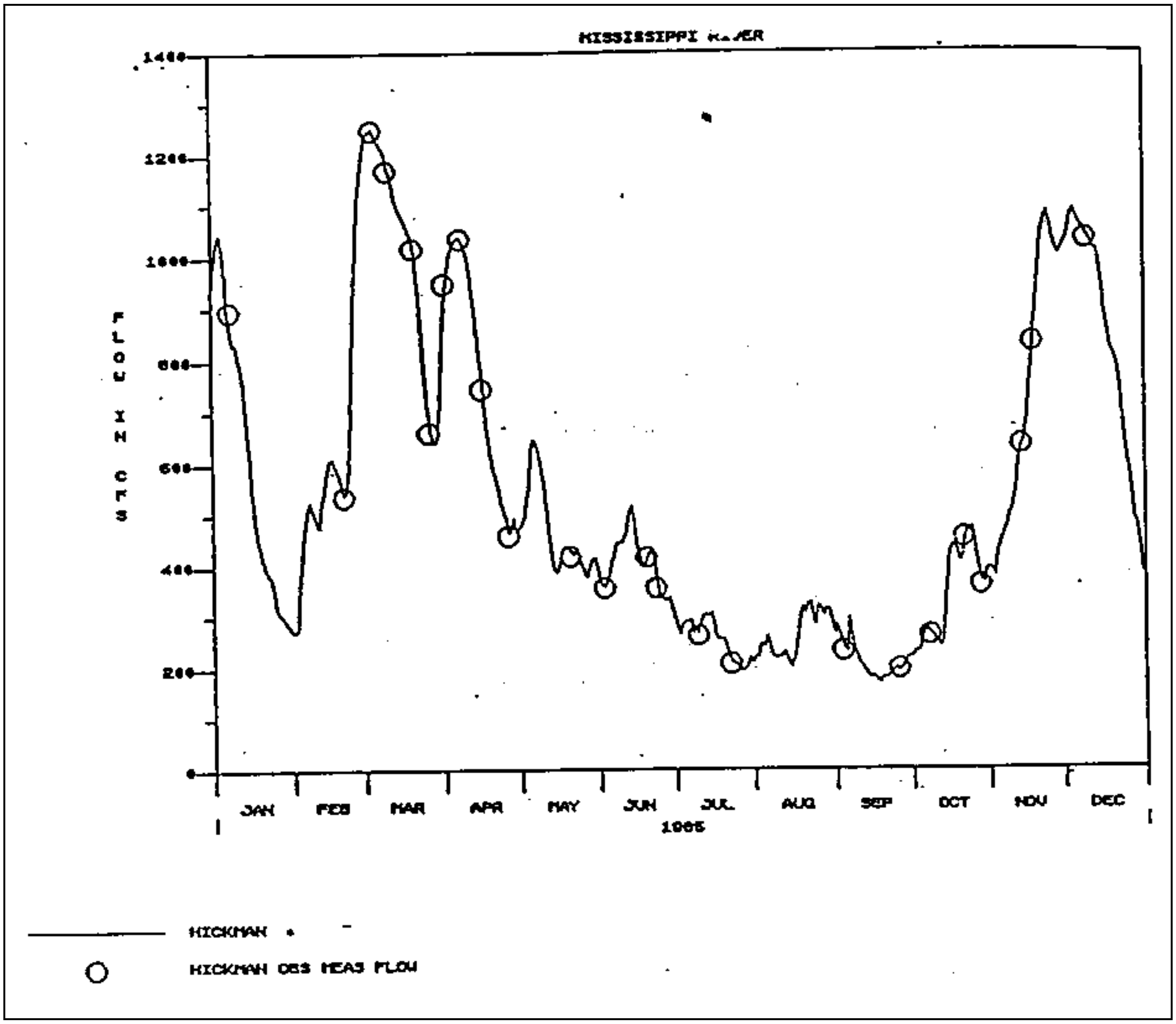

Figure 5-4. Computed versus observed flow at Hickman

both reasonable. Because of the variable friction slope, Manning's equation does display the looped rating curve; but the boundary condition still should be placed well downstream of the area of interest. Any model which uses Manning's equation as the downstream boundary condition should be tested for sensitivity to confirm that its use at the boundary has no affect on the area of interest.

c. Lateral inflow. Lateral inflow (or outflow for a diversion) data also constitute a boundary condition. Unlike upstream and downstream boundary conditions which translate into an independent equation, lateral inflow (q_L) augments the equations of continuity and momentum (see Equations 5-2 and 5-3). Lateral inflow can come from gaged and ungaged areas, and can be located at a point and/or uniformly distributed along the length of the valley.

(1) In any river system a part of the drainage will not be gaged. To provide an accurate and consistent simulation, the modeler must estimate the inflow from those ungaged areas. Along the Illinois River, for example, there is 2,579 square miles of ungaged drainage between the Marseilles and Kingston Mines gages, which is about 52 percent of the total gaged area. Figure 5-6 shows a simulation result at Kingston Mines without the ungaged drainage. The omission of the ungaged drainage produced a uniform error of about 1 foot in the simulated

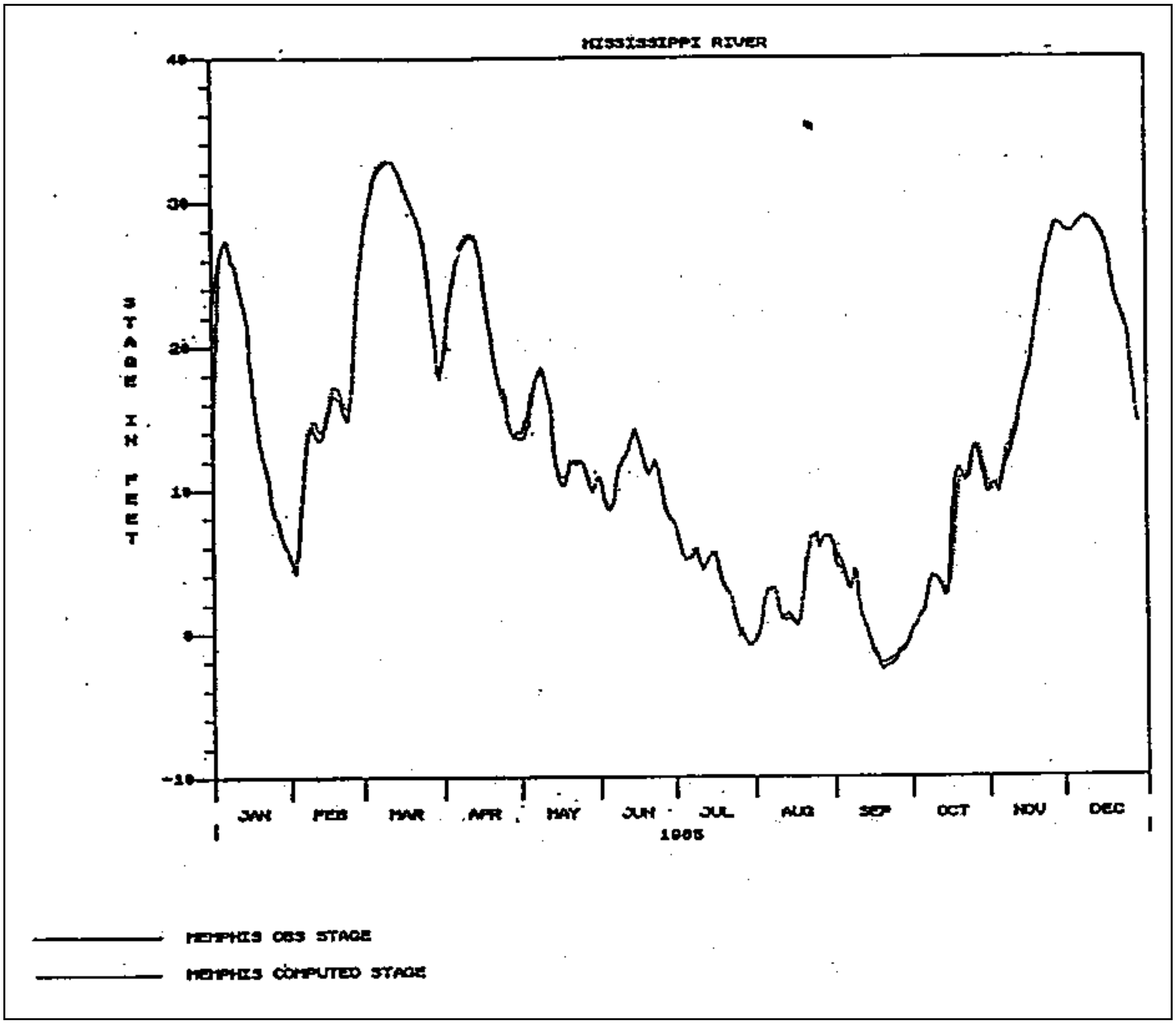

Figure 5-5. Reproduction of Stage at Memphis from Stages Routed from Cairo, 220 Miles Upstream

stage hydrograph. This difference could have been corrected by adjusting the *n* values, but the error would have become apparent as an inconsistency when verifying against other events. Figure 5-7 shows the correct simulation which includes the ungaged inflow.

(2) The estimation of ungaged inflow is difficult because of the wide variation in spatial rainfall patterns. Two methods are proposed: estimating runoff using drainage area ratios applied to gaged watersheds in the vicinity and use of a rainfall-runoff model. Drainage area ratios work well for large events when the rainfall is relatively uniformly distributed spatially. For smaller events, which cause small peaks in low flow, the method is less appropriate. A hydrologic model is preferable, but it may be an additional study step to develop and maintain, and requires precipitation data. Small, often unnamed, tributaries may be lumped together and entered uniformly as a single hydrograph which is distributed along a portion of the stream. Generally, the distribution is according to floodplain distance. Uniform lateral inflow is for the convenience of the modeler.

(3) Lateral inflow from a gaged tributary or from a large ungaged tributary is usually entered at a point. For streams with a flat bed slope a tributary inflow causes a disruption in the stage profile, as shown in Figure 5-8 by the correspondence between flow and stage discontinuities. Locating point inflows, even for ungaged areas,

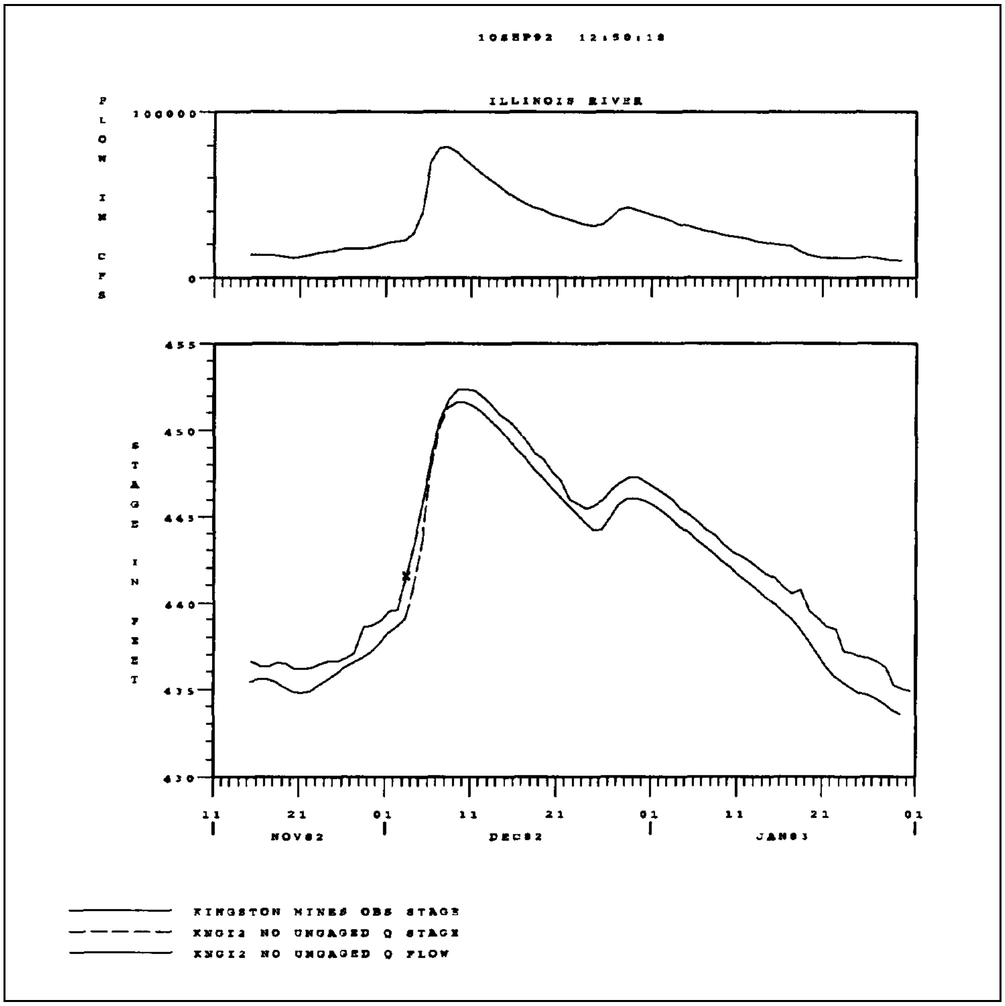

Figure 5-6. Simulation of the Illinois River at Kingston Mines without 2,579 square miles of ungaged drainage

may be a determining factor in the accuracy of the model. For the Illinois River, unsatisfactory results were produced if inflows from greater than 100 square miles were not entered at a point.

5-7. Steps to Follow in Modeling a River System

The following is a sequence of steps to follow when modeling a river system using unsteady flow. In subsequent sections, some of these topics will be expanded.

a. Prepare schematic diagram. The basic schematic diagram shows the layout of the river system and the principal tributaries for which gaged flow data are available. It is best to model every tributary for which cross-sectional data are available since the degree of detail determines the accuracy and consistency of the simulation. Also, tributaries can be modeled at modest additional cost in computer time and engineer effort. The scope of the model should be large enough so that errors in the downstream boundary condition do not affect results at the locations of interest. An example schematic diagram for the Red River of the North is shown in Figure 5-9 (U.S. Army Corps of Engineers 1990c).

b. Collect cross-sectional data. Collect all the cross-sectional data available on the main stem and tributaries. If data are old, or suspect for any reason, new data may be required. Usually cross section data are unavailable on all but the largest tributaries, thus limiting

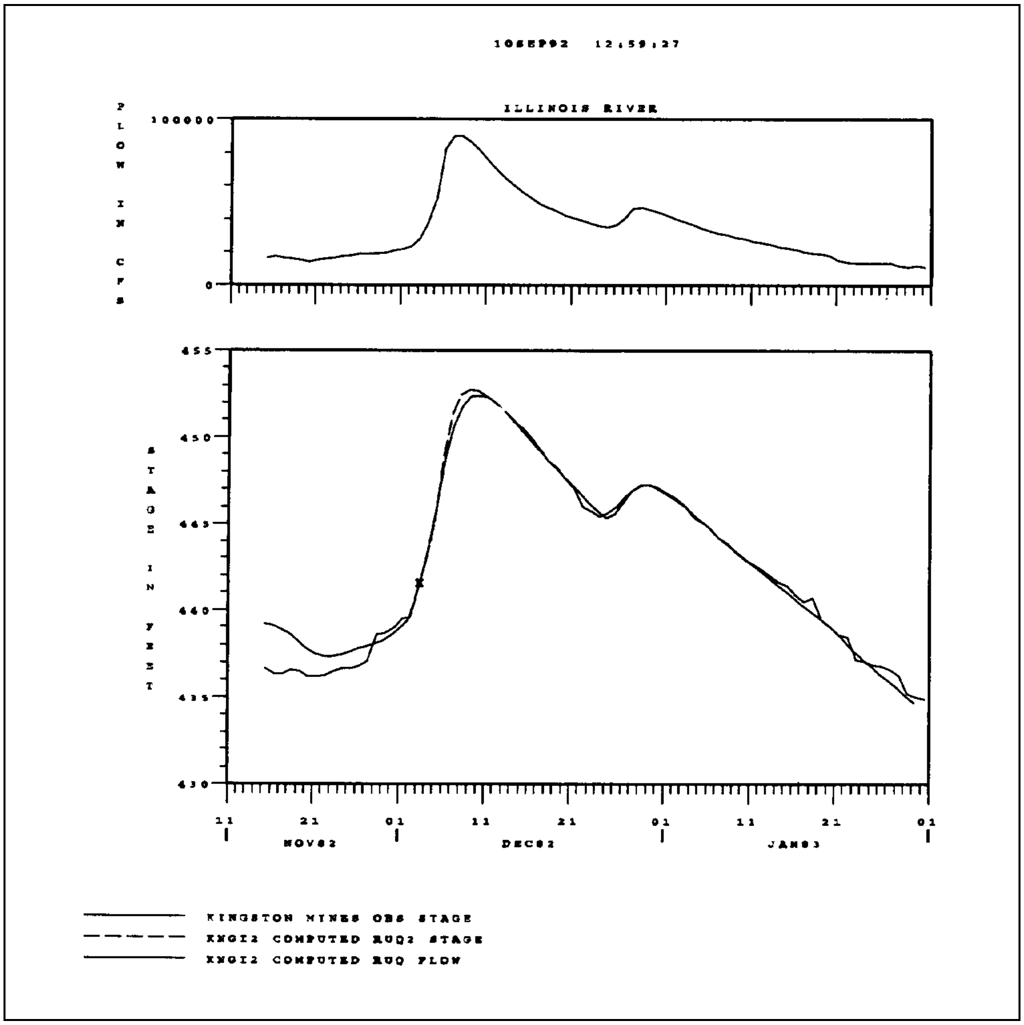

Figure 5-7. Simulation of the Illinois River at Kingston Mines including 2,579 square miles of ungaged drainage

the scope of the model. Study funds may limit the number of new surveys. If a major tributary has no surveyed cross sections, consider approximating the channel cross section and obtaining overbank information from USGS quadrangle maps. Remember, accuracy and consistency depend on the degree of detail. Details of cross section positioning are presented in Appendix D.

c. Collect stream gage data. Collect flow and stage data for the main stem and all tributaries. It is recommended that a data base such as HEC-DSS (U.S. Army Corps of Engineers 1990d) be used to organize observed data and maintain, display, and analyze computed results.

d. Develop gaging table. Develop a table showing all stream gaging locations from upstream to downstream, all major tributaries with gages, all major tributaries without gages, and reaches with uniform lateral inflow. For an unsteady flow simulation to be successful, every square mile of drainage must contribute inflow to the model. The gaging table locates the ungaged drainage and identifies the source from which ungaged inflow will be estimated. Table 5-1 is such a table for the Lower Mississippi River.

e. Revise schematic diagram. Revise the diagram by identifying all the reaches to be modeled, the locations of the gages, and all inflow points. To some extent, the gaging table and the schematic diagram are redundant, but the graphical display in the diagram helps assure an accurate definition of the system.

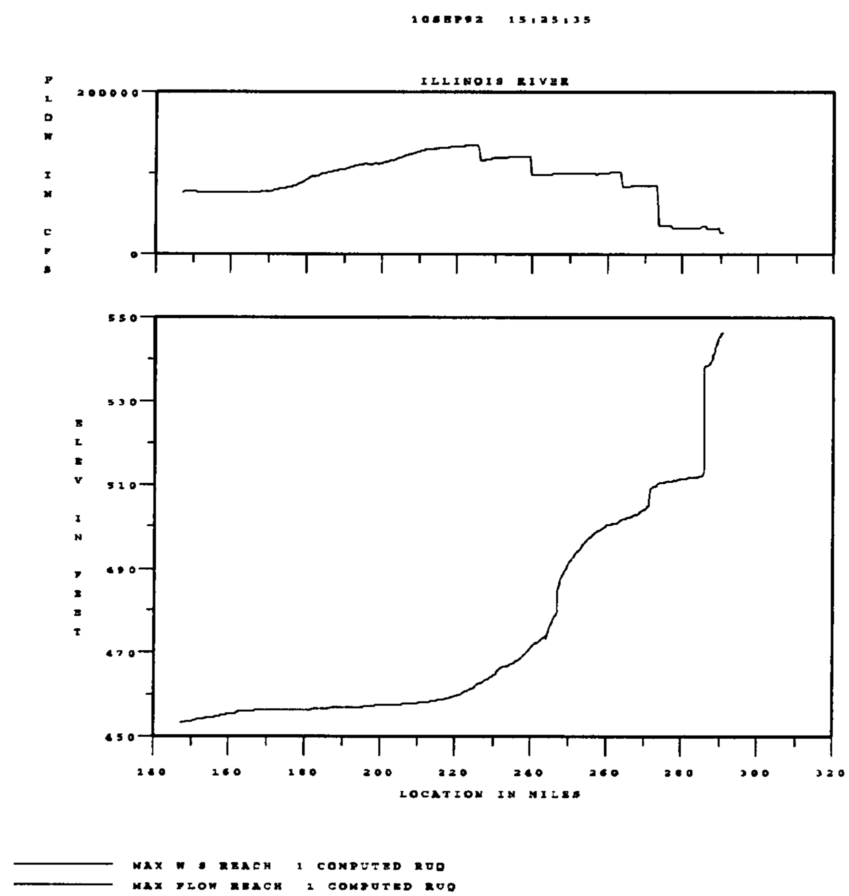

Figure 5-8. Disruption of the stage profile of the Illinois River by inflow from the Fox River

f. Assemble cross section file. On the basis of the schematic diagram, prepare the geometric data file. See Appendix D.

g. Identify a calibration event. Choose a time period that includes one of the largest events of record. The period should also include low flow and should contain the maximum amount of stage data.

h. Assemble boundary condition file. From the gaging table and the schematic diagram, assemble the boundary condition file locating all point and uniform lateral inflows in their proper locations.

i. Calibration. Calibrate the data to reproduce the calibration event.

j. Verification. Verify the simulation using other periods and events in the record. Minor adjustments to the parameters are acceptable, but no major changes should be needed. If the reproduction is inadequate, attempt to identify why.

5-8. Accuracy of Observed Data

All observed data are subject to measurement error. Both the operation and calibration of an unsteady flow model are based primarily on flow and stage data from gaging stations. Some stations have better records than others. It is the management of the error which results in the quality and consistency of the model. Consistency is the ability to reproduce multiple events with a single calibrated data set.

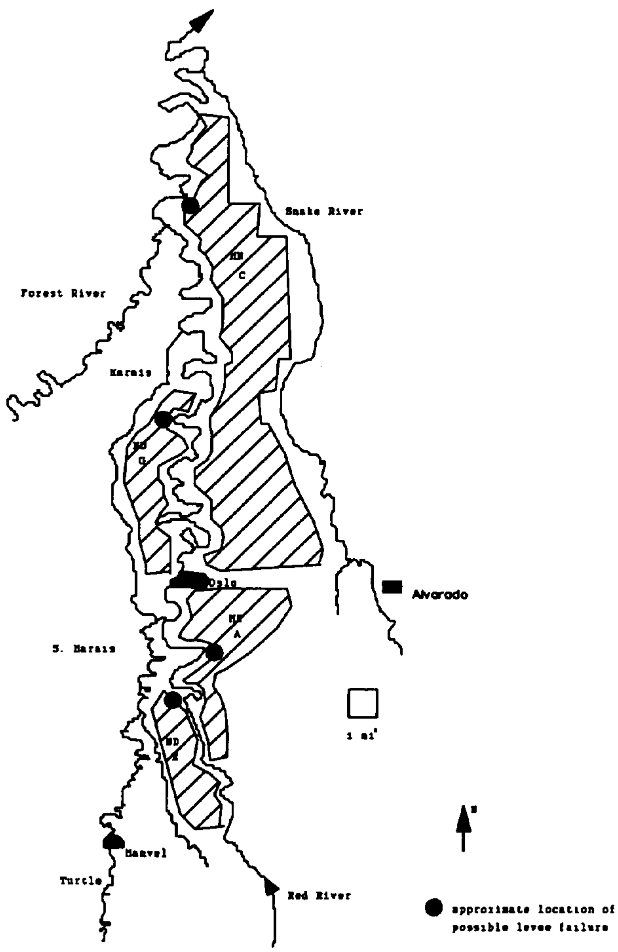

Figure 5-9. Schematic diagram for the Red River of the North

Table 5-1
Gaging Table for the Lower Mississippi River

MISSISSIPPI RIVER					TRIBUTARY DATA			
Mississippi River Gage	River Mile	Drainage Area (Sq. Mi.)	Gage Zero (Ft NGVD)	Ungaged Drainage (Sq. Mi.)	Tributary	Gaging Station	Drainage Area (Sq. Mi.)	River Mile
Chester, IL	109.9	708,563	341.05					
Thebes, IL	43.7	713,200	300.00	4637				
Birds Point, MO	2.0	713,397		197				
					Ohio River	L&D 52 (TW)	203000	
					Ohio River	Cairo	203040	953.8
Wickliffe, KY	951.5	917,400	269.12	963				
Columbus, KY	937.2	917,900	266.38	500				
Hickman, KY	922.0	918,500	264.73	600				
New Madrid, MO	889.0	919,200	255.48	700				
Caruthersville,	846.4	919,400	235.49	200				
Cottonwood Pt. MO	832.7	919,500	230.18	100				
					S. Fk. Deer R.	Halls, TN	1014	
					N. Fk. Deer R.	Dyersburg, TN	939	
					Obion River	Bogata, TN	2033	
					Obion River		3986	
Gage 158	819.1	924000	218.33	514				
Osceola, AR	783.5							
Fulton, TN	778.2	924,300	208.61	300				
					Hatchie R.	Rialto, TN	2308	773.3
Richardson, TN	769							
					Wolf River	Raleigh, TN	770	738.6
Memphis, TN	734.7	928,700	183.91	1322				
Star Landing, MS	707.4							
Mhoon Landing, MS	687.5	929,200	161.22					
					St. Francis Bay	Riverfront, AR	5141	672.4
					St. Francis R.	Parkin, AR	900	
Helena, AR	663.1	937,700	141.7	2459				
Fair Landing, AR	632.5	937,800	132.2					
					White River	Clarendon, AR	25497	599
Nr Rosedale, MS	592.1	965,800	108.73					
					Arkansas R.	Pine Bluff, AR	138000	581.4
Arkansas City, AR	554.1	1,104,360	96.66	560				
Greenville, MS	531.3	1,104,460	74.92					
Lake Prov, LA	487.2	1,104,560	69.71					
					Yazoo River	Yazoo City, MS	8900	
Vicksburg, MS	435.7	1,118,160	46.23					
St. Joseph, LA	396.4	1,122,660	33.12					
Natchez, MS	363.3	1,123,160	17.28					
Knox Landing, LA	313.7	1,124,700	0.00					
Red R. Landing,	302.4	1,125,000	0.00					
Bayou Sara, LA	265.4	1,125,400	0.00					
Baton Rouge, LA	228.4	1,125,810	0.00					
Plaquemine, LA	208.8	1,125,830	0.00					
Donaldsonville, LA	175.4	1,125,860	0.00					
Reserve, LA	138.7	1,125,880	0.00					
Bonnet Carre, LA	128.0	1,125,890	0.00					
New Orleans, LA	102.8	1,125,910	0.00					
Chalmette, LA	91.0	1,125,920	0.00					
West Pointe, LA	48.7	1,125,940	0.00					
Empire, LA	29.5	1,125,960	0.00					
Fort Jackson, LA	18.6	1,125,965	0.00					
Head of Passes, LA	-.6	1,125,970	0.00					

a. Stage data. Stage data are the most accurate type of hydrologic data. Stage measurement is accurate to within the amplitude of wind induced gravity waves and the consistency of the recording device. Experience has shown that gravity waves are typically about ± 0.1 foot in magnitude; see Figure 5-10. Traditional recording devices, e.g., strip chart recorders and paper tapes, which were predominant until the early 80's, tended to lose their accuracy with time. Each month, when the gage reader changed the tape, the automatic and the manual gage readings were recorded. Usually the difference was a couple of tenths of a foot although, occasionally, big discrepancies were found. The recorded readings were typically then adjusted by a linear relationship with time to match the manual reading under the assumption that the error increased gradually with time. The validity of this assumption may be questionable. These errors, which may be hidden, have bearing on how well the model seems to match observed data. Another problem is that gages sometimes lose their datum. Figure 5-11 shows a comparison of the Des Plaines River stages at Lockport with those at Brandon Road Pool, which is downstream. For 1974, Brandon Road is higher than Lockport; hence, the Des Plaines River appears to be flowing backwards. Which gage is correct?

(1) Newer gages have electronic recorders and transmit data via satellite. Still, the gages are subject to the similar losses of accuracy with time. Also, satellite transmissions are subject to large errors which appear as spikes in the time series. These spikes are easy to discern, but if they are input to a simulation they are disastrous.

(2) Finally, point observations, say the 07:00 reading, are often read from the hourly satellite time series. Since the data may be oscillating (Figure 5-10) is one point representative of the overall time series?

b. Flow data. Flow is usually a derived, not a measured quantity. Periodic flow measurements, using velocity meters, are initially used to define a rating curve

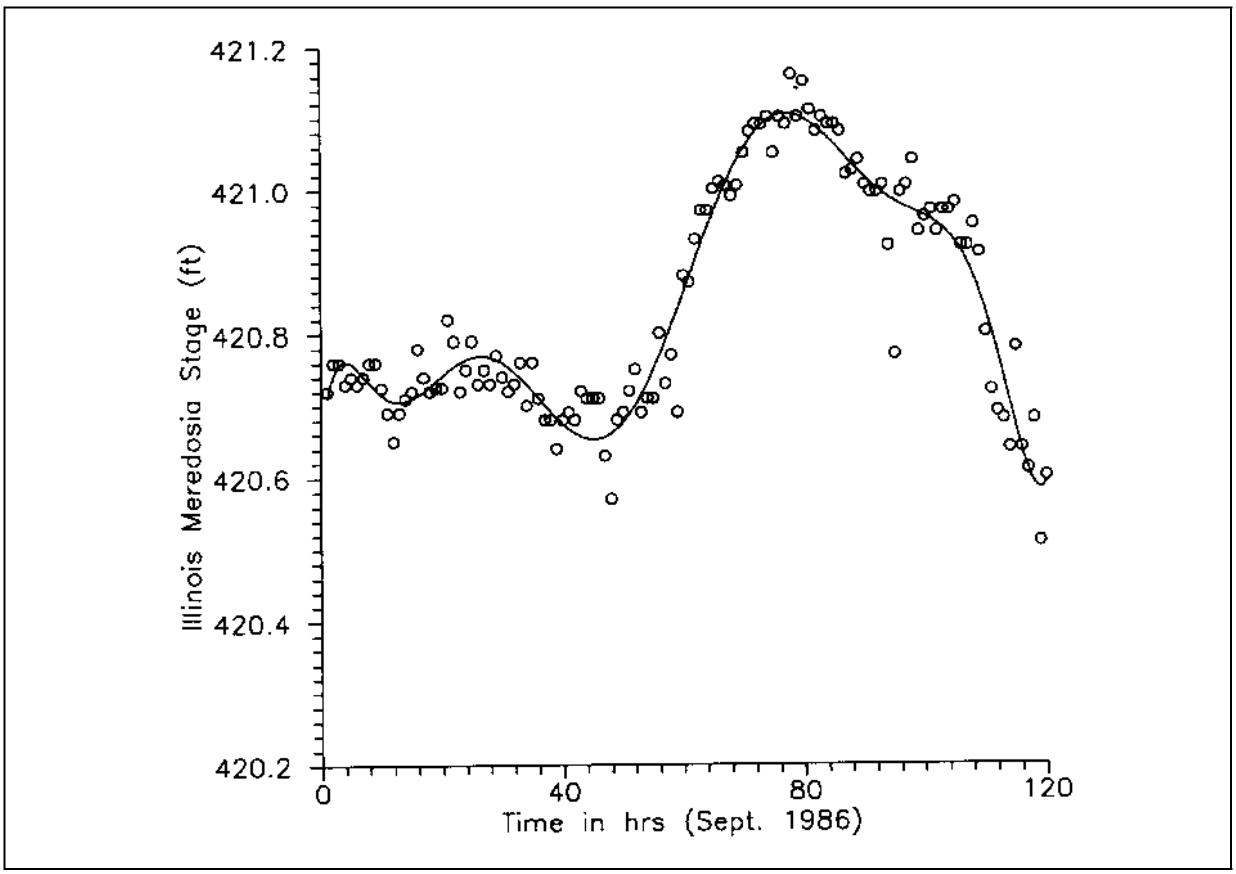

Figure 5-10. Oscillation of the 1-hour time series from a satellite for the Illinois River at Meredosia

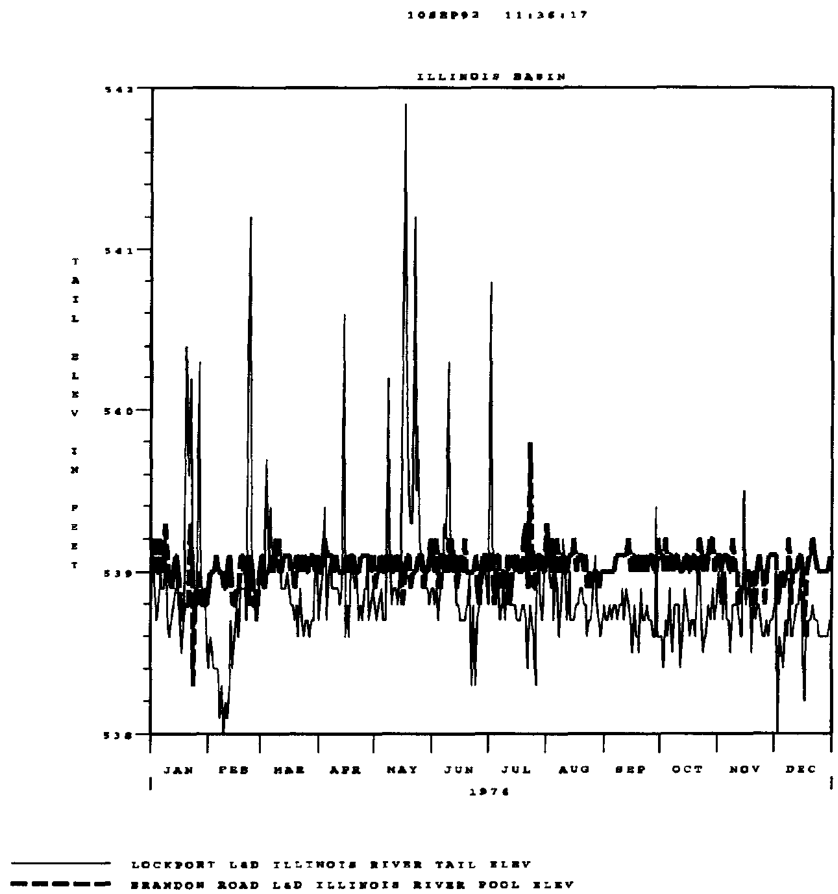

Figure 5-11. Stage hydrographs for the Illinois Waterway at Lockport Tailwater and Brandon Road Pool

and then to define shifts (seasonal, systematic, and random) from the rating curve. The "shifted" rating curve is then used to routinely derive discharge from stage with the discrete flow measurements being the only solid data.

(1) The USGS defines an "excellent" gaging station as having 95 percent of the daily discharges within ± 5 percent of the true value. The departure of the measurement from the rating curve is composed of the error in the measurement and the true shift. The shift is manually determined by attempting to isolate the error. The records at upland stations where the bed slope is large are usually good. On the other hand, the records on large rivers, where the bed slope is small and the dynamics are large, are suspect.

(2) The ability to adequately determine the rating curve shift depends on the frequency of discharge measurement. Long term trends of aggradation and degradation are adequately defined by even an infrequent measurement cycle. However, unless several measurements are taken during a flood event, it is unlikely that the loop or a seasonal shift will be adequately defined, resulting in an error. When modeling a river system, if a gaging station is used as an upstream boundary, the error results in an inconsistency in calibration between events which cannot be reconciled. On the Middle Mississippi River a base flow error of ± 5 percent results in a model inconsistency of ± 1 foot. If the lack of definition of the loop is added to the base error, sizable inconsistency can be explained. Slope stations are gaging stations which

are influenced by backwater. At these stations the rating curve is modified not only by the shift but also by a slope correction which is computed from the observed fall to a downstream gage. Discharge records at a slope station are seldom very good and should be used as boundary conditions with caution.

(3) There are gaging stations whose records are not very reliable. These are usually on streams with a flat bed slope or a mobile boundary. At these locations, only the actual flow measurements can be used with confidence.

5-9. Calibration and Verification

When a model is calibrated, the parameters which control the model's performance, primarily Manning's *n* and reach storage, are determined. The key to a successful calibration is to identify the true values of the parameters which control the system and not to use values that compensate for shortcomings in the geometry and/or the boundary conditions. Because unsteady flow models reproduce the entire range of flows, they should be calibrated to reproduce both low and high flows.

a. Manning's n. In the unsteady flow models used in the United States, the friction slope is generally modeled using Manning's equation. Manning's *n* value relates the roughness of the stream boundary to the friction force exerted on the system. For most problems, an initial estimate of Manning's *n* (it is only an educated guess) is used at the start of the calibration. The initial values are then adjusted to match observed stage data. When no observed stage data exists, the estimated values take on a greater importance since they are assumed to be representative of the system. See Appendix D for detailed information on selecting *n* values.

b. Calibration. Calibration of an unsteady flow model is a four step process. In the first step the *n* values are adjusted to reproduce the maximum stages of an event. The storage in the cross sections is then adjusted, if necessary, to improve timing and attenuation. In the third step, the flow versus Manning's *n* relationship is adjusted to reproduce both high and low flow event stages. Finally, the model is fine tuned to reproduce a longer period which should include the initial calibration event.

(1) The initial calibration event should be one of the larger events which are available in the time series. The purpose of this phase is to adjust the initial *n* values to match the crest of the event at all stations in the model. Figure 5-12 shows the hydrographs for the Illinois River at Havana after the initial calibration. Note that, although the crest stage is approximately correct, the timing of the hydrograph and the reproduction of low flow are deficient.

(2) Total storage as defined by river cross sections is almost always deficient. In natural rivers, the timing of the hydrograph is determined by storage and the dynamics of the flood wave. Timing can be adjusted by modifying storage, friction, and distribution of lateral inflows. If the timing cannot be calibrated by reasonable adjustment of these factors, then there is some other problem, most likely an error in the cross sections. For the Illinois River, which is confined by levees in the reach near Havana, an increase in overbank storage of about 20 percent yields the results shown in Figure 5-13; an increase in storage of about 40 percent yields those shown in Figure 5-14. Both changes are only minor increases in storage area because the overbanks are confined by levees.

(3) By varying Manning's *n* with flow the reproduction of stage is improved; see Figure 5-15. The model still does not reproduce the initial time steps, but the disagreement is probably caused by the initial conditions.

(4) The final calibration consists of fine tuning the flow-roughness relation and the adjustments in storage. The event selected should be an extension of the event chosen for the initial calibration. For the Illinois River example, the final calibration was performed for the period from 15 Nov 1982 to 15 Sep 1983. The event includes high flow and low flow and a second major flood in May 1983. Figure 5-16 shows the reproduction of stage at Havana during the period. The model parameters required only slight adjustment to better simulate low flow.

c. Verification. The calibrated model should be verified against two or more periods which include significant events. The periods should be long, approaching one year, so that seasonal effects can be detected. Figure 5-17 shows the reproduction of the 1974 observed data on the Illinois River.

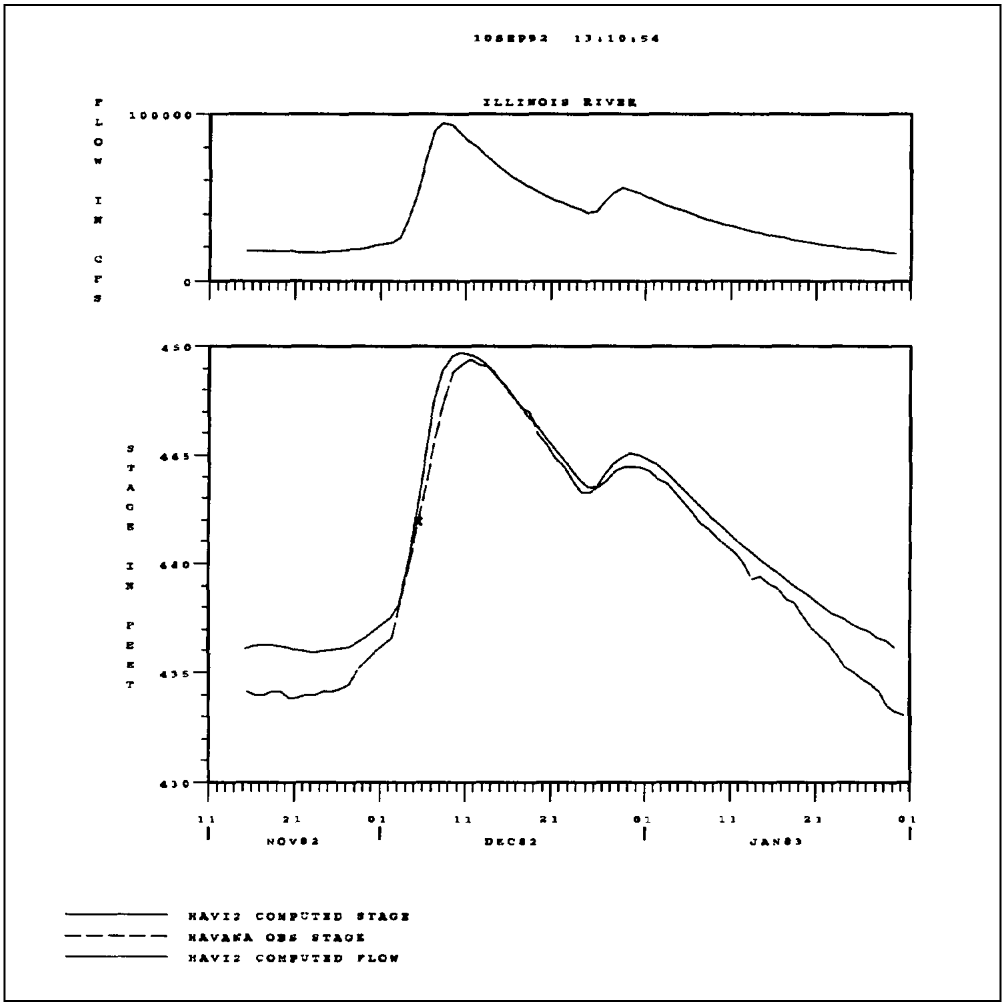

Figure 5-12. Hydrographs for the Illinois River at Havana after initial calibration

5-10. Example Applications of Unsteady Flow Models

Numerous applications, in addition to those presented above to illustrate the use of unsteady flow models, have been performed; the following is a brief summary. The one-dimensional unsteady flow program DWOPER, developed by the National Weather Service, has been used to simulate flood wave movement through the Central Basin of the Passaic River in New Jersey. This was a complex routing problem because of flat gradients and flow reversals that were involved (U.S. Army Corps of Engineers 1983). The one-dimensional unsteady flow model UNET has been applied to a 90-mile long reach of the Red River of the North to improve analysis of flooding on this river. The study reach was characterized by agricultural levees and other flow controlling features on a wide, flat floodplain (U.S. Army Corps of Engineers 1990c). Cunge et al. (1980) present several examples of applications to complex natural river systems. A study of potential mudflow movement in Castle Creek, near Mount Saint Helens was performed (U.S. Army Corps of Engineers 1990e) using the NWS DAMBRK model (Fread 1988).

Section II
Theory of Routing Models

5-11. Introduction

a. General. This section describes, in a one-dimensional context, the physical characteristics of flood waves passing through a reach of channel. An overview of

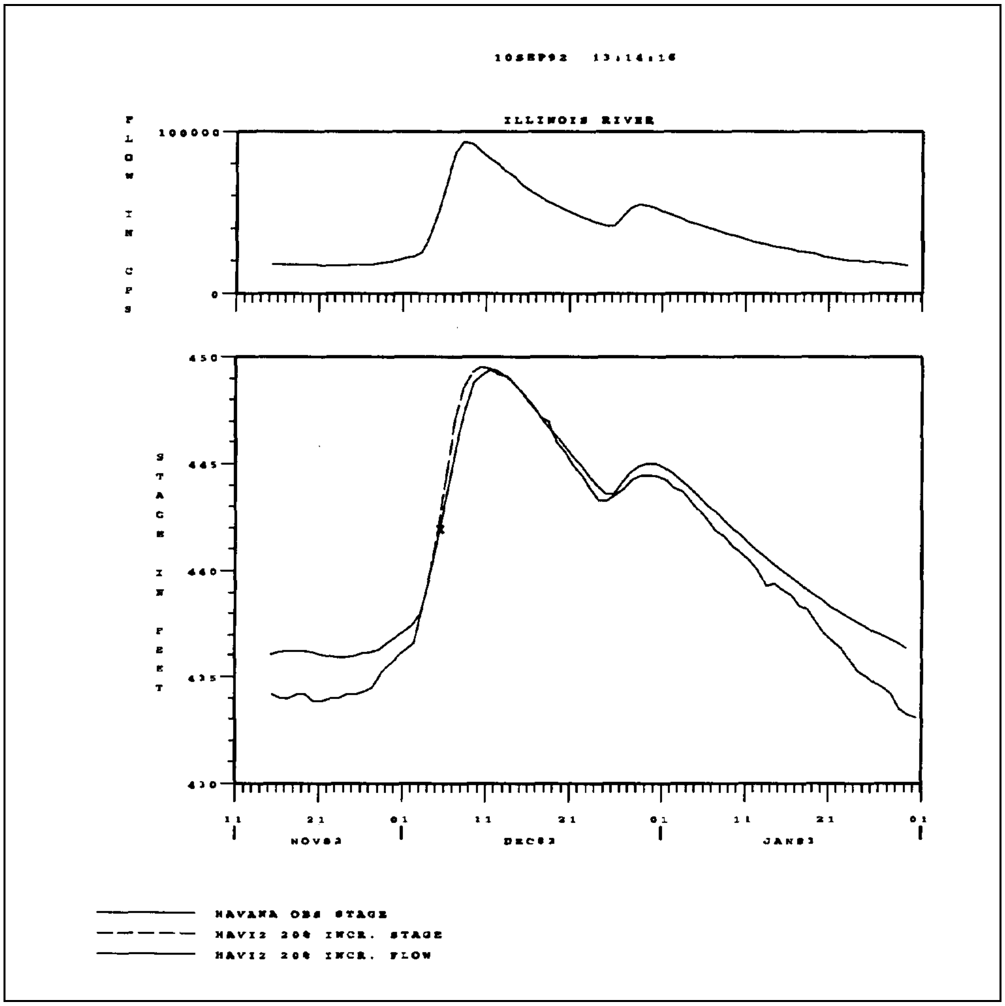

Figure 5-13. Hydrographs for the Illinois River at Havana with overbank storage increased by 20 percent

prediction techniques is presented: first hydraulic techniques, which simulate the wave motion by solving the mathematical equations governing the unsteady flow in the reach, and then hydrologic techniques, which compute outflow hydrographs directly from predetermined reach characteristics and a given inflow hydrograph. The effects that the assumptions characterizing a model have on its applicability are discussed.

b. Hydrologic routing versus hydraulic routing. In the nineteenth and early twentieth centuries, the approaches used to analyze problems associated with the movement of water were fragmented among different professions in accord with the area of endeavor affected by the particular case of water motion. The assumptions developed to allow solution of these complex problems varied widely in the different fields in accord with the inventiveness of the researcher and were generally unrelated. Classical hydrodynamicists studied the mathematics of potential flow of a perfect fluid, which water under certain circumstances imperfectly imitates. Mathematicians studied laminar flow, a turbulence-free phenomenon in which fluid mixing takes place only on a molecular level. Laminar flow is rarely seen in rivers; the high Reynolds numbers and boundary roughness of a typical river make turbulent flow the norm. Hydraulic engineers developed empirical formulas for head loss in turbulent flow in pipes. Because of the greater complexities of open channel flow, engineers devised assumptions and computational schemes to be as simple as possible for analyzing river flows.

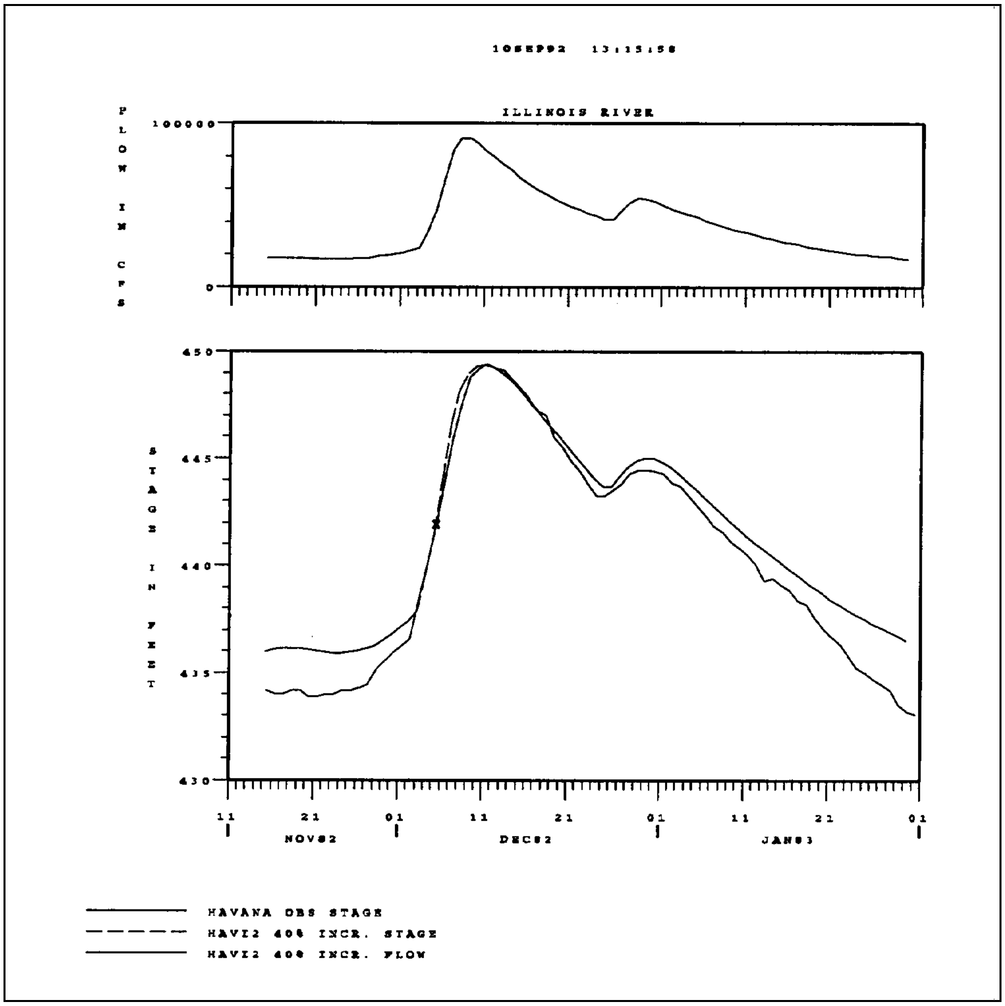

Figure 5-14. Hydrographs for the Illinois River at Havana with overbank storage increased by 40 percent

(1) This section seeks to relate the so-called hydrologic and hydraulic approaches to flood routing. The hydrologic approaches, which are simpler to use but harder to defend theoretically, are viewed from the point of view of the hydraulic approaches, which are better grounded in basic theory but relatively difficult to apply.

(2) The aim of both approaches is the same: to determine the response in a given reach of a watercourse to a given inflow sequence (usually a flood hydrograph), and, both recognize the physical principle of conservation of mass. They both seek to account, at all times, for all of the volume of water initially in the stream and that of the inflow(s) and outflow(s). The volume of water stored in a reach varies with time as a flood wave passes through.

(3) Mathematically, with *I(t)* representing an inflow sequence (hydrograph), *T(t)* the net lateral inflow along the length of the reach (tributary inflow minus infiltration, etc.), *O(t)* the outflow hydrograph, and *S(t)* the volume of water (storage) between the inflow and outflow sections, the principle of conservation of mass can be written:

$$I(t) + T(t) - O(t) = \frac{dS(t)}{dt} \tag{5-1}$$

(4) The argument, *t*, is explicitly stated to underscore the premise that the equation holds true at each instant of time. With the inflow hydrograph given, and with the tributary hydrograph given, estimated, or

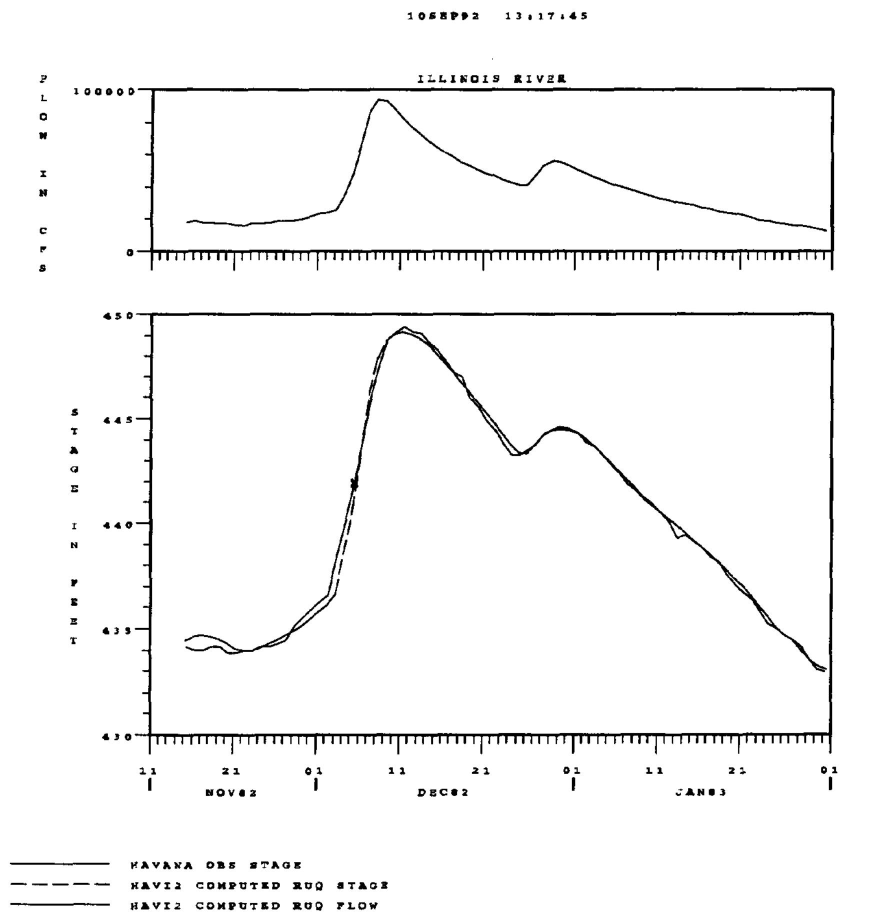

Figure 5-15. Hydrographs for the Illinois River at Havana with adjusted flow-Manning's *n* relationship

neglected, the outflow hydrograph can be computed if the relation of the storage to the hydrographs is also known. It is on this issue, the relationship between the geometrical quantity, storage, and the kinematic quantities, discharge hydrographs, that the hydrologic and hydraulic approaches differ.

(5) The hydrologic techniques focus attention on discharge hydrographs. The outflow discharge hydrograph constituting the response of the reach to the inflow hydrograph is computed directly, and after that is done, the water levels in the reach are somehow related to the discharges. To achieve such a direct solution for the outflow hydrograph, a storage versus flow relation is assumed, either empirically on the basis of flood events of record for the reach, or theoretically on the basis of some simplifying physical assumption. In the most empirical of the hydrologic techniques, the storage is not even considered; inflow hydrographs are manipulated by an averaging technique flexible enough to allow matching of computed and measured outflow hydrographs.

(a) Furthermore, in hydrologic methods, the study reach is treated as a whole. Even if the reach is broken into subreaches, as some of the techniques propose, it is assumed that the outflow hydrographs can be determined sequentially, from upstream to downstream. The outflow hydrograph of one subreach serves as the inflow hydrograph for the neighboring downstream subreach.

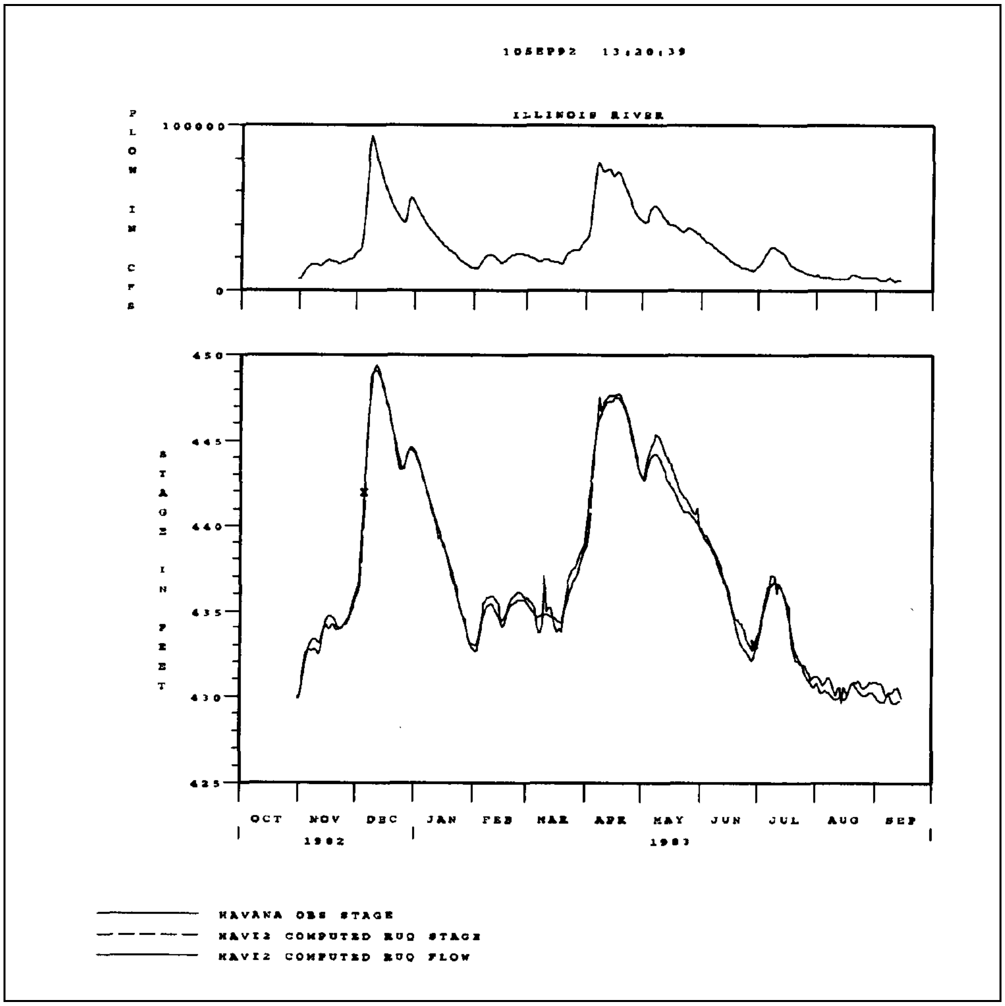

Figure 5-16. Hydrographs for the Illinois River at Havana with flow-Manning's *n* relationship adjusted to reproduce the 1983 flood

(b) In this approach, time is the only variable, and the mathematics of the simulation reduce generally to an ordinary differential equation. This equation relates the sought after time variation of the outflow to the given time variation of the inflow and to the given response characteristics of the reach, e.g. a storage versus flow relationship. The hydrologic techniques typically solve this differential equation numerically, i.e. algebraically, through the use of finite-sized time steps.

(6) The hydraulic approaches explicitly recognize, in addition to the physical principle of mass conservation, a second physical principle, one or another form of conservation of momentum. If, then, an assumption is made regarding the shape that graphs of the variation of stage and discharge along the reach would have, absolute values for both profiles can be found. The usual assumption is that the shape of the stage and discharge profiles cannot be given *a priori* for the reach as a whole. It must be broken into a sufficient number of distance steps so that the shape of depth and discharge variation in each can be assumed to be a straight line. For this reason, the hydraulic techniques generally require a determination of depth and discharge at a sequence of stations within the reach, even if the conditions are in fact sought at only one point.

(a) As a result, a characteristic feature of hydraulic approaches is the calculation of flow variables in the interior of the study reach, even if they are not of special interest. For example, to arrive at the outflow

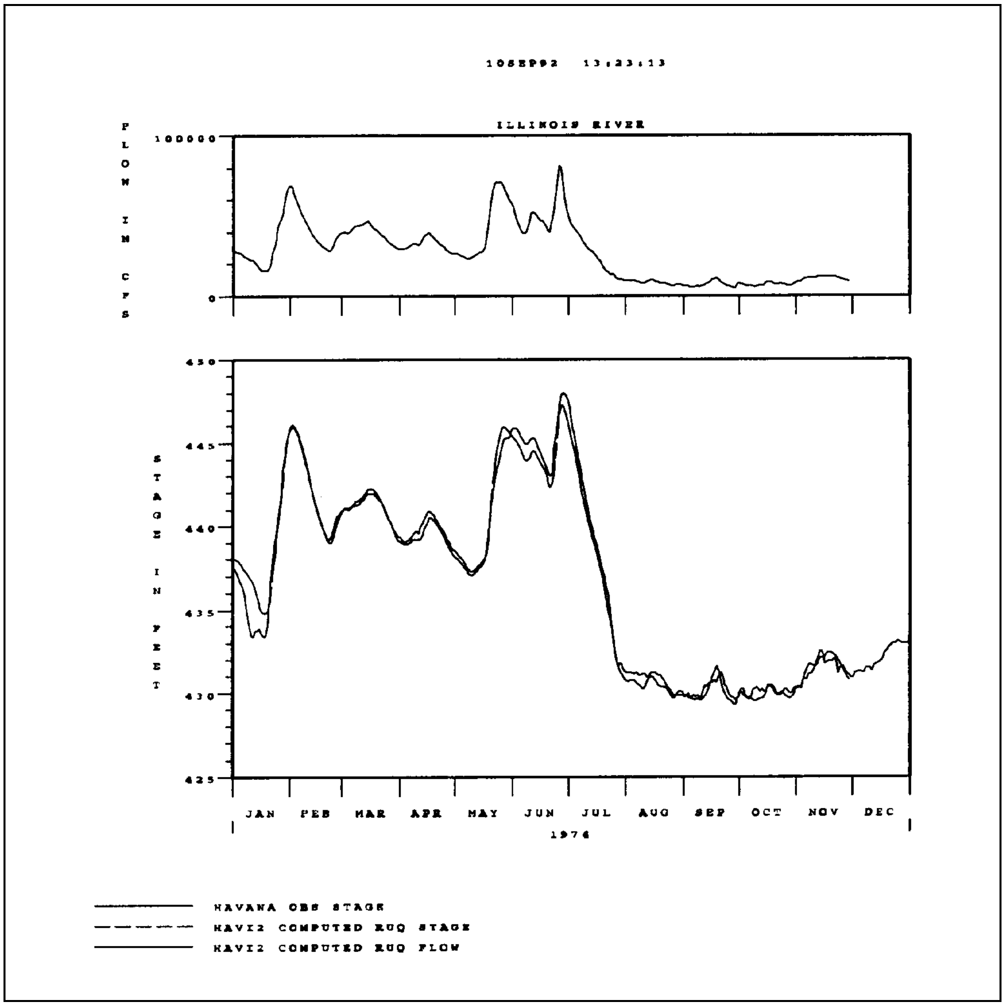

Figure 5-17. Verification of the Illinois River model against 1974 observed data

hydrograph for a reach subject to a given inflow hydrograph at its upstream end, the hydraulic methods compute water surface elevations and discharges at a sequence of stations in the interior of the reach. The desired hydrograph is computed along with all interior hydrographs, and stages in the reach are routinely determined as part of the solution. In another example, the calculated advance of a dam-break flood wave is a by-product of calculations of flow conditions in the interior of the wave.

(b) In the limit, as the number of distance steps increases indefinitely, while the size of each is correspondingly reduced, the governing physical principles lead to partial differential equations in distance along the channel and time. The dependent variables are the time dependent profiles of depth and discharge (or depth and discharge hydrographs at all stations in the reach). These partial differential equations are generally solved numerically, algebraically, in finite-sized distance, and time steps with the aid of high-speed electronic computers.

(7) The hydrologic techniques are often easier to apply than the hydraulic techniques and are usually associated with quicker, less troublesome, computations. Hydraulic methods require a description of the geometry and roughness of the reach usually defined by cross sections and reach lengths. Those hydrologic methods which use past flood hydrograph records to estimate the response of the reach bypass such detailed analysis of the physical characteristics of the reach; the lumped effect of its physical characteristics is assumed to be incorporated

into the measured responses. And if, in fact, the reach does behave sufficiently like the calibration events for the flood being studied, the hydrologic approach may be nearly as accurate as any of the hydraulic routing schemes for determining discharge. The difficulty, of course, is in establishing the storage versus flow relation pertinent to the subject flood.

5-12. Unsteady Flow Model

a. Unsteady flow equations. Derivations of the unsteady flow equations are presented in numerous references. Chow (1959), Fread (1978), and User's Manual for UNET (U.S. Army Corps of Engineers 1991b)] are three of such references. They can be obtained from the two-dimensional equations presented in Chapter 4 by assuming that the dependent variables only change in one direction, x, and that direction is along the river axis rather than being a cartesian coordinate. Common formulations of the equations are as follows:

Equation of continuity

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + \frac{\partial S}{\partial t} = q_L \qquad (5\text{-}2)$$

Equation of momentum

$$\frac{\partial Q}{\partial t} + \frac{\partial (QV)}{\partial x} + g\,A\left(\frac{\partial z}{\partial x} + S_f\right) = q_L\,V_L \qquad (5\text{-}3)$$

where

Q = flow
A = active flow area
S = storage area
q_L = lateral inflow per unit flow distance
V = Q / A = average flow velocity
g = acceleration of gravity
z = water surface elevation
S_f = friction slope
V_L = average velocity of the lateral inflow
x = flow distance
t = time

(1) The assumptions implicit to the unsteady flow equations are essentially the same as those for the steady flow equations: (a) the flow is gradually varied; that is, there are no abrupt changes in flow magnitude or direction; (b) the pressure distribution is hydrostatic; therefore, the vertical component of velocity can be neglected. This means, for example, that the unsteady flow equations should not be used to analyze flow over a spillway, and (c) the momentum correction factor is assumed to be 1.

(2) The magnitude of each of the terms in the momentum equation plays a significant role in the hydraulics of the system. The terms in equation 5-3 are:

$$\frac{\partial Q}{\partial t} = \textit{local acceleration}$$

$$\frac{\partial (QV)}{\partial x} = \textit{advective acceleration}$$

$$\frac{\partial z}{\partial x} = \textit{water surface slope}$$

$$S_f = \textit{friction slope}$$

The water surface slope can be expressed as

$$\frac{\partial z}{\partial x} = \frac{\partial h}{\partial x} - S_o \qquad (5\text{-}4)$$

in which h is the depth and

$$\frac{\partial h}{\partial x} = \textit{pressure term}$$

$$S_o = \textit{bed slope}$$

The roles of these terms are discussed below.

b. Weaknesses of the unsteady flow equations.

(1) Friction slope is the portion of the energy slope which overcomes the shear force exerted by the bed and banks, and it cannot be measured. To quantify the friction slope, the Manning or Chezy formulas for steady flow are used:

Manning's Equation

$$S_f = \frac{Q|Q|n^2}{2.21A^2R^{4/3}} \qquad (5\text{-}5)$$

where

n = Manning's n value
R = hydraulic radius

Chezy's Equation

$$S_f = \frac{Q|Q|}{A^2 C^2 R} \tag{5-6}$$

in which C is the Chezy coefficient. Note the use of the absolute value of discharge; this keeps the sign of S_f proper for flow reversals.

(2) Equations 5-5 and 5-6 are semi-empirical equations for steady flow, but they also produce acceptable results for unsteady flow. Other equations have been proposed for estimating the friction slope Einstein (1950), Simons and Sentürk (1976), and ASCE (1975). Typically, these equations are logarithmic and contain sediment parameters. Most modelers have avoided these equations because they are computationally inconvenient, requiring an iterative solution to solve for the friction slope within each time step.

c. Force exerted by structures. Bridge piers, embankments, dams, and other hydraulic structures exert a force on the flow which is not considered in the momentum equation presented above. To illustrate this force, consider submerged flow over a broad crested weir as shown in Figure 5-18. The unequal pressure distribution on the upstream and downstream faces exerts a net force in the upstream direction on the flow. This force is not included in the friction term, nor is it included by the pressure force from the bank which is included in the water surface slope term. If the force is not included in the momentum equation, the computed swell head upstream of the structure will be too small. Moreover, the force is seldom quantified. The emphasis of research has been to quantify the energy loss through structures, which is useful for computing the swell head for steady flow.

(1) Modelers Fread (1978), and Barkau (1985) have proposed augmenting the momentum equation with an additional slope term based on the energy loss:

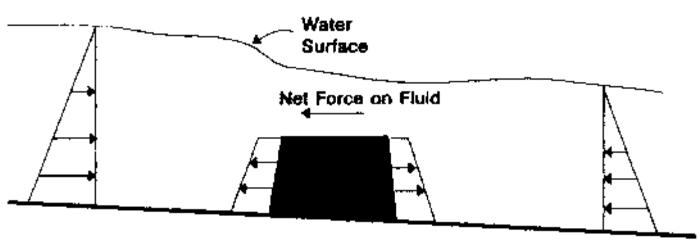

Figure 5-18. Exterior forces acting on a control volume of fluid flowing over a broad crested weir

$$S_h = \frac{h_L}{dx} \qquad (5\text{-}7)$$

in which h_L is the head loss due to the force and dx is the distance over which the loss occurs.

(2) Since energy loss is obtained by integrating force applied over distance, Equation 5-7 estimates an additional energy slope to overcome the force. The added slope produces the correct swell head upstream of the structure. The augmented momentum equation now becomes:

$$\frac{\partial Q}{\partial t} + \frac{\partial (QV)}{\partial x} + g\,A\left(\frac{\partial z}{\partial x} + S_f + S_h\right) = q_L\,V_L \qquad (5\text{-}8)$$

d. Subcritical and supercritical flow. The unsteady flow equations are wave equations. Disturbances propagate according to the rate

$$\frac{dx}{dt} = V \pm c \qquad (5\text{-}9)$$

where

c = the celerity of a gravity wave
c = $(gD)^{1/2}$
D = hydraulic depth

(1) If $V < c$, the flow is subcritical, and disturbances move both upstream and downstream. Hence, a disturbance downstream, such as a rise in stage, propagates upstream. If $V > c$, the flow is supercritical, and the velocity sweeps all disturbances downstream. Hence, a stage disturbance downstream is not felt upstream.

(2) Equation 5-9 has profound implications for the application of the unsteady flow equations. Subcritical flow disturbances travel both upstream and downstream; therefore, boundary conditions must be specified at both the upstream and downstream ends of the routing reach. For supercritical flow, the boundary conditions are only specified at the upstream end.

(3) Near critical depth, the location for the boundary conditions is changing; hence, the flow and the numerical solution may become unstable. Instability when the depth is near critical is one of the greatest problems encountered when modeling unsteady flow. Most streams which are modeled with unsteady flow are subcritical at higher stages but, at lower stages the pool and riffle sequence usually dominates flow. Supercritical flow can occur at the riffles. Because unsteady flow models simulate the full range of flow, the models can become unstable during low flows.

e. Numerical models. An unsteady flow model (also called a dynamic wave model) solves the full momentum and continuity equations. Forces from all three sources (gravity, pressure, and friction) and the resulting changes in momentum (local and advective accelerations) are all explicitly considered along with mass conservation. If the assumption of one-dimensional flow is justified, and the discretization of flow variables introduces little error, then the simulation results are as accurate as the input data. Unsteady flow models differ in their underlying physical assumptions, in the way in which the real continuous variation of flow variables with space and time is approximated or represented by discrete sets of numbers, and in the mathematical techniques used to solve the resulting equations. Other differences reflect the range of different steam networks, channel geometries, control structures, or flow situations that the model is designed to simulate. For example, not all dynamic wave models are equipped to handle supercritical flow; a typical indication of failure is oscillating water surface profiles and an aborted execution. There are also differences (which can strongly effect study effort) in input data structure, user operation, documentation, user support, and presentation of results.

(1) Such a model can accurately simulate flows in which acceleration plays an important role, such as flood waves stemming from sharply rising hydrographs such as a dam break flood; disturbances of essentially still water, for example the drawdown of water in the reservoir behind a ruptured dam; and seiching, which is a long period longitudinal oscillation of water. Another example of a situation that can be modeled only by a dynamic wave model is the reflection of a dam break flood wave from a channel constriction.

(2) As the bed slope becomes small, it becomes less important than the water surface slope and the acceleration terms play a greater role. The looped rating curve is an example of this phenomenon. For streams on a low slope, the rising limb of the hydrograph passes at a lower stage than the falling limb for a particular discharge. The loop for the Illinois River at Kingston Mines during the December 1982 flood is shown in Figure 5-19. The flow and stage hydrographs were shown in Figure 5-8.

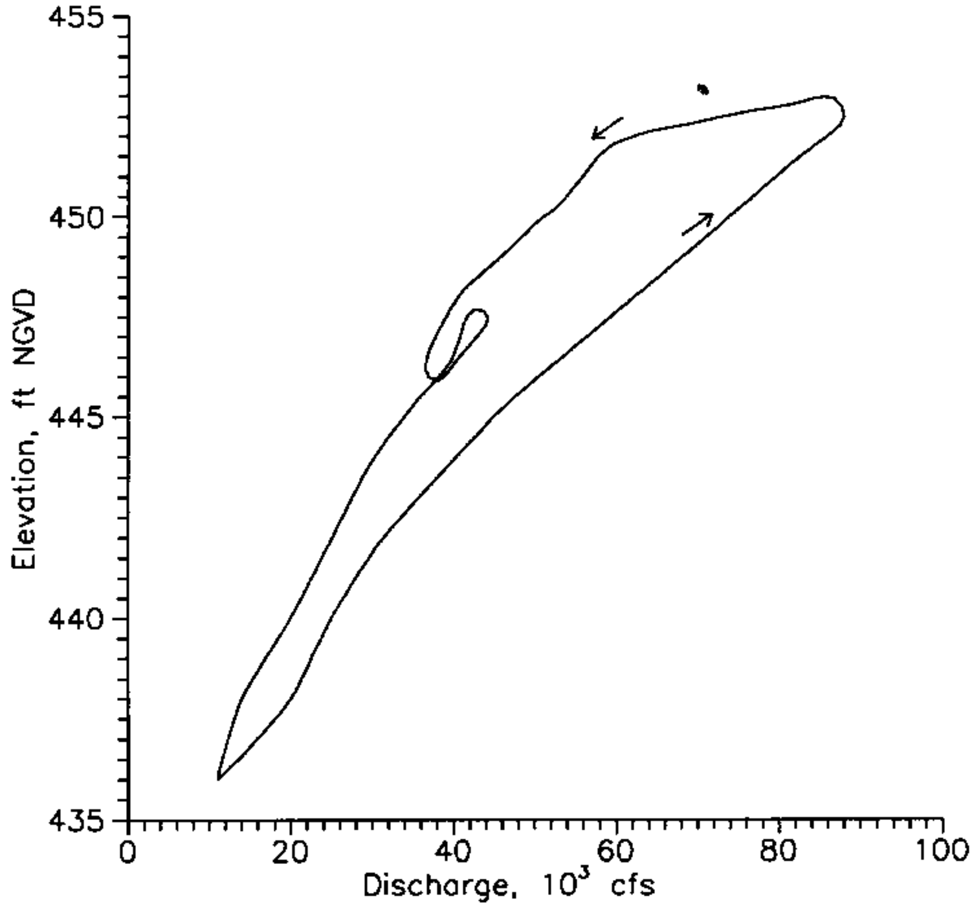

Figure 5-19. Looped rating curve for the Illinois River at Kingston Mines, 15 Nov 82 to 31 Jan 83

The peak flow always precedes peak stage. The loop can be explained with the help of Figure 5-20. The slope of the water surface is greater on the rising limb than on the falling limb, thus the flow is accelerating on the rise and decelerating on the fall.

(3) If the flow changes rapidly, then the acceleration terms become important regardless of the slope of the bed. The advective acceleration term diffuses the discharge downstream; it lengthens and attenuates any rapid change in discharge. Figure 5-21 shows a test of routing a rapidly rising and falling hypothetical hydrograph through a channel of unit width using an unsteady flow model. In 8,000 feet the peak discharge had attenuated by over a third and the hydrograph had lengthened dramatically. This is typical of dam break type waves.

f. Numerical approximations. Discretization, the representation of a continuous field of flow by arrays of discrete values, is a major concern in the construction of unsteady flow models. The choice of scheme influences the ease of writing, correcting, and modifying the program; the speed at which the program executes; accuracy of the solution, including satisfaction of volume conservation, momentum conservation, and computation of proper wave velocities; the robustness of the model; and ultimately, its stability.

(1) Explicit solution schemes allow the computation of flow variables at the end of a time step at one point in the channel, independent of the solution for neighboring points. Implicit schemes solve for the flow variables at the end of a time step at all points in the channel simultaneously. The former are easier to program and maintain, but require small time steps to avoid computational instability. The required size of the time steps for usually much less than that needed to resolve the rates at

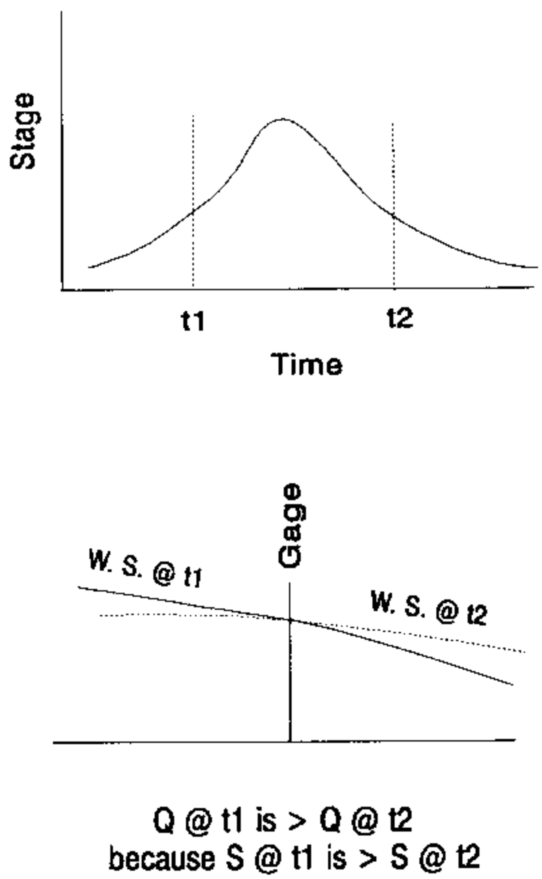

Figure 5-20. Explanation for looped rating curve effect

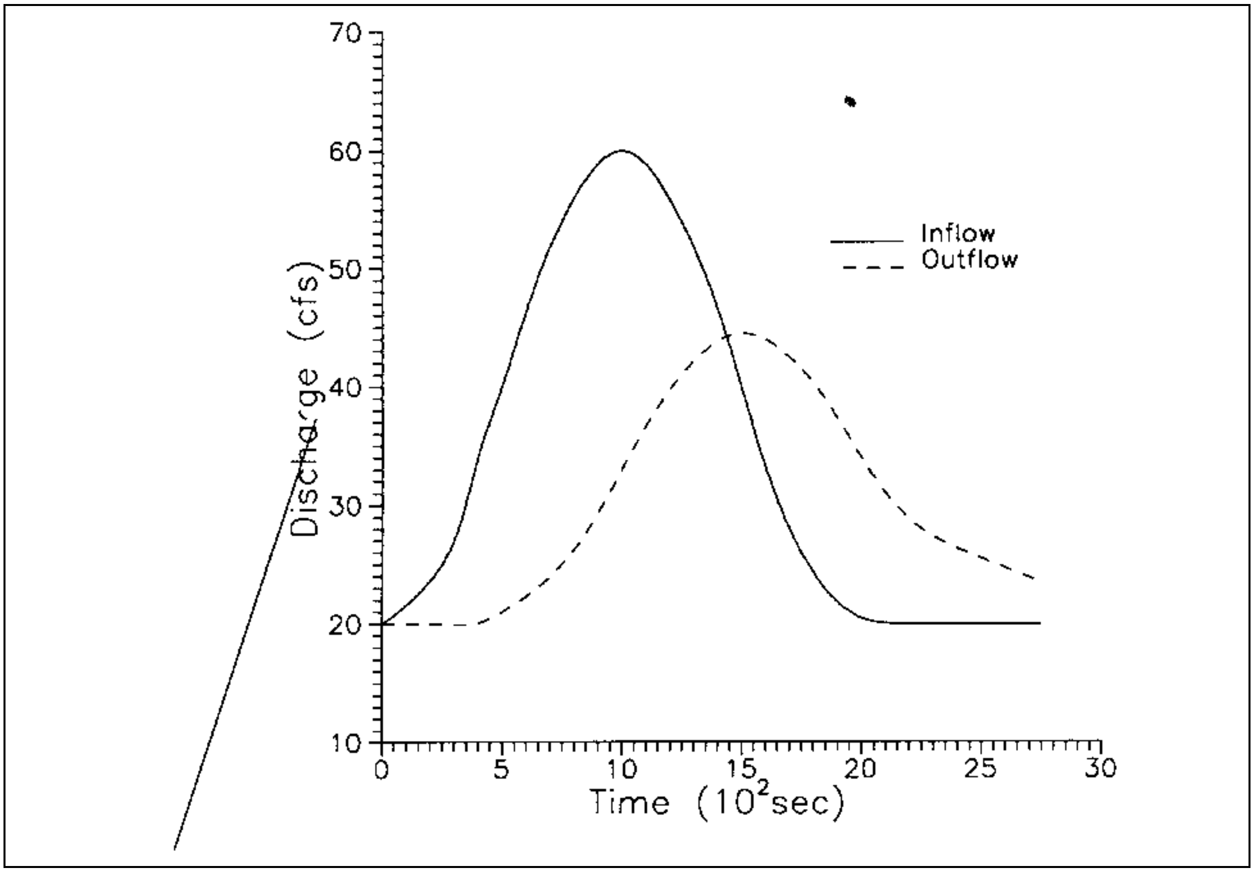

Figure 5-21. Attenuation from a hypothetical dam break type flood routed 8,000 feet downstream through a channel of unit width

explicit schemes is which changes are occurring to the flows at reach boundaries. This can lead to a very inefficient solution. The time steps for implicit schemes are, theoretically, dependent only on accuracy criteria and can be many times larger than in explicit schemes. Implicit models appear, further, to be generally more robust.

(2) Most of the successful models available today use an implicit finite difference scheme (Fread 1978, 1988; Shaffranek et al. 1981; Johnson 1982; U.S. Army Corps of Engineers 1991b).

5-13. Diffusion Model

For some flow conditions the water surface slope and the friction slope are nearly equal and the momentum equation becomes

$$\frac{\partial z}{\partial x} \approx -S_f \qquad (5\text{-}10)$$

This is the diffusion wave, or zero-inertia approximation. Forces from all three sources are assumed to be in equilibrium, so that the acceleration is zero. If the sum of local acceleration (a measure of unsteadiness) $\partial Q/\partial t$ and advective acceleration (a measure of nonuniformity) $\partial(QV)/\partial x$ is small compared to the sum of weight (i.e., gravitational) and pressure components, this model is capable of producing a simulation virtually as realistic as the dynamic wave model. This is often the case for flows at a low Froude number.

a. Assumptions. Local and advective accelerations are often of similar magnitude and opposite sign; their sum is typically smaller than either one alone.

b. Nonuniformity. Only when the nonuniformity of the flow is primarily the result of nonuniform channel geometry, rather than because of unsteadiness, can the local acceleration be small compared to advective acceleration.

(1) The neglect of all acceleration terms in the diffusion model has advantages and disadvantages. A major advantage is a more robust model, because the acceleration terms are sometimes the source of computational fragility, especially in a near-critical or supercritical flow. To a diffusion model, all flows are infinitely subcritical.

(2) The disadvantages include the inability to simulate certain kinds of flow, seiching is infinitely damped, and bores are imperfectly imitated by relatively gradual rises in water surface elevation.

(3) The magnitude of the error in outflow hydrograph prediction for typical inflow hydrographs depends on the channel and inflow hydrograph characteristics.

5-14. Kinematic Wave Model

a. Slope. If the slope of the bed is relatively steep and the change in discharge is moderate, the pressure term and the acceleration terms become small compared to the bed and friction slope terms. Hence, the friction slope and the bed slope are approximately in balance as shown in Equation 5-11.

$$S_f \approx S_o \tag{5-11}$$

This is called the kinematic wave approximation, and the flow can only be routed downstream. The water surface elevation at each section can be calculated from Manning's equation or obtained from a single-valued rating curve for any discharge. There are no backwater effects. The physical assumptions in this approximate method are often justified in overland flow or steep channels if the flow is well established so that there is little acceleration.

b. Limitations.

(1) The method is patently useless in horizontal channels, because there is drag but no streamwise weight component. It typically overestimates water depth in channels of small slope. As a rule of thumb, the kinematic wave approximation may be applicable for slopes greater than 10 feet per mile, depending upon the shape of the hydrograph. Experience has shown that kinematic wave should not be used when analyzing flows in rivers.

(2) A characteristic feature of flood wave behavior computed with this method is that, in the absence of lateral inflow/outflow, there is no subsidence of the crest. Certain numerical schemes introduce a spurious numerical subsidence, but that cannot be used rationally to model real subsidence. The phenomenon of kinematic shock allows flood wave subsidence within the context of kinematic wave theory, but does not model real subsidence. When subsidence is important, a kinematic wave model should not be used.

(3) The major advantage of kinematic wave is that it displays no computational problems at critical depth.

5-15. Accuracy of Approximate Hydraulic Models

Numerical criteria are presented in Ponce (1989) for estimating the relative accuracy of approximate models. Some of the criteria are based on the relative magnitude of neglected terms in the unsteady flow equations (5-3 and 5-4). Others, dealing with hydrologic methods, are concerned with subreach length relative to length of the flood wave. Still others stem from the results of comparative tests.

a. Kinematic versus diffusion. According to Ponce (1989), kinematic and diffusion wave models may be used in reaches with little or no downstream control. The diffusion wave has a wider range of applicability than the kinematic wave and should be used unless a strong case can be made for the latter. Ponce suggests the following criteria for determining applicability of the two methods:

The kinematic wave model can be used if

$$\frac{T_r S_o u_o}{d_o} > 85 \tag{5-12}$$

The diffusion wave model can be used if

$$T_r S_o \sqrt{\frac{g}{d_o}} > 15 \tag{5-13}$$

where

T_r = hydrograph time of rise
S_o = equilibrium energy slope (or bottom slope for channel of regular cross section)
u_o = average velocity
d_o = average flow depth
g = acceleration of gravity

b. Data requirements. These depend on the nature of the method and are described in the sections which follow and in Appendix D. In general, hydraulic models require channel geometry, boundary roughness, the initial state of the water in the channel, and an upstream flow hydrograph.

(1) An upstream boundary condition with its time variation, such as a discharge or depth hydrograph, must be specified, as must be the tributary inflows or outflows. In the special case of supercritical flow at the upstream end of the reach, both depth and discharge must be given to a dynamic wave model.

(2) With the dynamic wave and diffusion models, either a depth or discharge hydrograph is required at the downstream end. In the special case of supercritical flow at the outlet (dynamic wave model), no downstream boundary condition can be given.

(3) No downstream condition can be given to the kinematic wave model, nor to any of the hydrologic models, as they all employ "marching" solutions, progressing from upstream to downstream.

5-16. Muskingum-Cunge Model

While the origin of this model is the Muskingum method, a hydrologic technique, its theoretical basis and application, typically to a large number of subreaches, suggest that its classification be as a hydraulic method. As such, it is a subset of the diffusion approach; the additional assumption, linearization about normal depth at the local discharge, leads to problems with accuracy at low values of bottom slope and precludes analysis of flows in which backwater effects play a role. Its advantages over the diffusion approach are not known at this time; comparisons might prove it to be a more robust model.

5-17. Hydrologic Routing Schemes

Hydrologic routing focuses on the study reach as a whole; there is still need for two equations to solve for the two related variables, water surface elevation and discharge, even if these are required at just one location. The principle of mass conservation supplies one of the required equations but, instead of applying the momentum equation in the interior of the flow, a different theoretical or empirical relation provides the second equation. A summary discussion is presented below.

a. Average-lag methods. Two significant features of flood hydrographs have long been observed in many rivers. Reflecting the wave-like character of flood behavior, hydrographs at successive stations are displaced in time; peaks, for example, occur later at each successive downstream station. In other words, downstream hydrographs lag upstream hydrographs. The second observation is that, usually, hydrograph peaks exhibit subsidence; that is, a decrease in peak value with distance downstream if there is no significant tributary inflow.

(1) Such behavior is observed in the results of the so-called average-lag methods, empirical techniques based on averages of inflow hydrograph values lagged in time. Averages of groups of hydrograph values are always less than the largest of the group unless all members of the group are equal; in particular, the average of values in the vicinity of the peak will be less than the peak itself. Freedom in choosing the time spacing of points on the inflow hydrograph, the number of points to include in the average, the weighting coefficients defining the average, the number and length of subreaches to which to successively apply the technique, and the travel time for the hydrograph in each subreach; i.e., the amount of time to lag the hydrograph, often provides enough flexibility to allow a match of lagged average reach-outflow hydrographs with observed ones in a calibration event. Many years of familiarity with a reach of river and with the observed hydrographs can facilitate choosing the parameters of such a method for a reasonably good fit of computed and measured hydrographs, but satisfactory routing under different circumstances would have to be considered fortuitous. There are many ways in which hydrograph values can be averaged and lagged. There is no theoretical reason to favor one over another.

b. Progressive average-lag method. This technique as found in EM 1110-2-1408 also known as Straddle-Stagger (U.S. Army Corps of Engineers 1990a), is the most empirical of these methods. It provides hydrographs which exhibit subsidence and time lag, and these can often be made to match observed hydrographs through adjustment of the arithmetic parameters of the method.

(1) The reach is treated as a whole; subreach length equals reach length. Equal weight is given to the inflowing hydrograph values in determining their average. The time period over which averaging occurs is centered on the inflow value being routed; i.e., the one at a lag-time duration earlier than the time pertinent to the outflow hydrograph value. The constant time interval used to define the inflow hydrograph, the number of points used for averaging, and the lag time (outflow value time minus routed inflow value time, expressed as an integer number of time intervals) are chosen by trial and error for a best fit with observations.

(2) The hope in using this method is that the storage/hydrograph relation that exists for the reach in the calibration event is reflected in the arithmetic parameters determined, and that these will continue to be valid for the subject event. The lack of any theoretical basis for this hope makes the method unreasonable rather than approximate. The term approximate suggests that there is some control over the amount of error. But, in principle, the error in the computed subsidence for the subject event could be zero, plus or minus a hundred percent or more. Only if a series of calibration events lead to about the same parameter values in each case could one reasonably suppose that a subject event in the same reach with about the same inflow hydrograph as the calibration events, calculated with those values of parameters, would yield an outflow hydrograph of about the same accuracy as the calibration events. In general, the method is not recommended.

c. Successive average-lag method. In this technique (EM 1110-2-1408 1960), also known as the Tatum Method, each ordinate of the outflow hydrograph for a subreach is the numerical average of the routed inflow value and the preceding ordinate in the hydrograph. The ordinates of the inflow hydrograph are separated by constant time intervals, Δt, a parameter of the method. Subreach length is defined as the distance traveled by the flood wave in a time interval $\Delta t/2$, taken as the lag time. The outflow hydrograph for a subreach constitutes the inflow hydrograph for the next subreach, for which the procedure is repeated.

(1) Additional subreaches are introduced until the outflow for the subject reach has been determined. The number of subreaches constitutes another parameter of the method. The parameter values are chosen for a best fit with calibration hydrographs.

(2) A physical interpretation of the Tatum Method exists; it corresponds to a linear Modified Puls technique in which subreach storage is directly proportional to subreach outflow with the constant of proportionality $K = \Delta t/2$. Nonetheless, the method, like Progressive Average-Lag, must be considered empirical, and is not generally recommended.

d. Modified Puls. This approach is more rational than the average-lag methods, because it strives to solve the mass-conservation relationship (equation 5-2) by providing a second, storage versus flow, relation necessary to close the system.

(1) The method is characterized by a far-reaching physical assumption which, unfortunately, is often not warranted in rivers. The required storage versus flow relation stems from the assumption that there exists a unique relationship between storage in the reach and outflow from the reach. It is further assumed that this relationship can be found for the reach, either theoretically or empirically from past events; and that, once determined, applies to the study event. The mathematical form of the relationship is not important, a graph or table of numbers will suffice.

(2) An empirical relation can be found by measuring discharges as they vary with time during a calibration flood event at the inlet and outlet of the reach and applying the volume-conservation principle, (Equation 5-2). To the extent that tributary flow is accounted for, the relationship is valid for the event for which the information was recorded. To the extent that the relationship will continue to be valid for another event, or a different inflow hydrograph, it can be successfully used to predict outflow hydrographs for that event.

(3) A storage-outflow relation can be easily devised for a channel which is so large that the water surface remains level during the event to be simulated (a reservoir or "level pool") and if a discharge coefficient, theoretical or empirical, is available for the outlet. This is the physical circumstance for which the basic assumption of the Modified Puls method is valid.

(4) Hypothetical relationships between storage and outflow are sometimes derived for rivers from steady flow computations. Steady water surface profiles and, hence, water volumes, are computed in the reach for a sequence of discharges (outflows). The resulting table of

volumes as a function of discharge constitutes the storage/outflow relation. Such a relation ignores the effects of unsteadiness on the flood wave profile and hence on storage. The method can be successful if the local accelerations are negligible; i.e., if the reach is so geometrically nonuniform that advective accelerations from that source are large, and, at the same time, the rate of rise of the flood is so small that local and advective accelerations resulting from the unsteadiness are negligible in comparison.

(5) A potential source of major error with the Modified Puls method is that, in some flow circumstances, there is no physical relation between reach storage and outflow. The method does not account for the time changes in water flow that are transmitted as waves and not instantaneously from one end of the reach to the other. For example, a sharp increase in discharge at the upstream end of a reach produces a wave of increased depth that travels downstream at some velocity, generally somewhat greater than the water velocity. Thus, the storage in the channel starts to increase immediately, but the outflow is not affected at all until the wave finally arrives at the downstream end of the reach.

(6) The storage/outflow relation derived from a sequence of steady flows is unique; it plots as a single curve without hysteresis. But even a stage/outflow relation at a gaging station exhibits hysteresis in unsteady flow, with one branch of the hysteresis loop describing the function for the rising limb of the hydrograph and the other for the falling limb. This is due to the influence of local acceleration and its effect on water surface slope and advective acceleration. While a small amount of hysteresis is not of great concern, the hysteresis loop for a storage/outflow relation can be markedly more pronounced because of the traveling flood wave volume passing through the reach.

(7) In order to devise a more correct theoretical relation between storage and outflow than is possible using the entire reach as a unit (typically, the shape of the water surface within the reach is unknown), the reach may be broken into a number of subreaches. In each of these, the water surface is assumed level, or parallel to the bottom, and the outflow of a subreach is related to the depth through some uniform flow formula such as the Manning equation. As the number of subreaches is increased indefinitely, the scheme approaches that of the kinematic wave theory.

(8) Except for level-pool routing, the Modified Puls method should be used with caution, particularly for conditions outside the range of events used for calibration.

e. Muskingum technique. The assumption is made that the storage in a reach at some instant is related to both the inflow and outflow of the reach at that instant, which is more realistic than relating storage to outflow alone, as in the Modified Puls method. In the Muskingum technique storage is assumed to be in part directly proportional to inflow and in part directly proportional to outflow. The constants of proportionality can be determined either empirically from a study of known events or theoretically as in the Muskingum-Cunge technique. The major cause for concern in empirical derivations is that the subject simulation event may not produce the same wave profiles as the calibration event(s).

f. Muskingum-Cunge technique. In addition to the diffusion wave assumptions, the assumption is made that during the passage of the flood wave down the reach, departures from normal depth in the reach are not great. Then the proportionality constants in the Muskingum method can be determined theoretically. The diffusion equations are linearized about normal depth for some average condition in the reach and the results manipulated to yield the proportionality coefficients. The theoretical nature of the determination of the coefficients suggests that this is a hydraulic rather than hydrologic technique, especially, if the reach is broken up into a large number of subreaches to account for the unknown shape of the flood wave and to better schematize the boundary geometry. It is also discussed in section 5-16.

g. Working R and D method. This method is the same as the Muskingum method in that storage is assumed to be related to both inflow and outflow, but not necessarily proportional. Tabulated or graphed relations are envisioned. The method has more potential than Modified Puls (which can be considered a subset of the working R and D method) because it allows for the possibility that reach storage depends on inflow as well as outflow.